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A Novel Framework for Cost Constrained Network Sharing

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Abstract—Network sharing is widely accepted as a cost effective approach for mobile network deployment. It remains uncertain, however, how regulators will evaluate network sharing agreements (NSA) for future networks in the context of the current competition law. For example, 5G mobile network operators (MNOs) seeking to enter NSAs may risk legal challenges, as regulators have not given MNOs sufficient guidance for self-evaluation of their NSAs. One way for MNOs to reduce the risk of legal challenge is to avoid sharing variable costs in the NSA. However, constraining costs to be non-variable (i.e., fixed) rules out the use of most pricing mechanisms that have been widely adopted for dynamic resource trading between MNOs. In this article, we propose a network sharing framework to allow dynamic resource sharing without the use of resource pricing. To incentivize sharing without pricing, our framework presents sharing as a means for MNOs differentiate services and better compete in the service market for profit. We evaluate our framework in a duopoly market model and demonstrate the economic and regulatory viability of our framework.

Index Terms—network sharing, 5G, beyond 5G, network economics, regulation

1 Introduction

1.1 Motivation

Network sharing is widely recognized as a key enabler towards fast and cost effective deployment of future networks [1]. In light of these benefits, regulators are warming up to network sharing. However, there are ongoing concerns that cooperation by operators at the network level can restrict competition at the retail level, as reiterated in recent statements by the European Commission.[1] Amplifying these concerns is a decade-long wave of mergers in the mobile service market [4], which have consolidated market power among a few mobile network operators (MNO) and increased the risk that future network sharing agreements (NSAs) harm retail competition. In response, MNOs have stated their openness to carefully design their agreements in order to comply with competition law and quickly navigate the regulatory approval process [5]. Unfortunately, authorities have not given MNOs sufficient guidance in identifying when an NSA breaches EU competition law [6], [7]. This lack of guidance has arguably lead to regulatory challenges following the formation of agreements. Hence, MNOs may feel the need to conservatively design NSAs to cautiously avoid regulatory grey-areas, or otherwise risk more legal challenges, which can result in slowed 5G rollout, fines, or loss of investment.[2]

For operators seeking to conservatively design their NSAs, some limited guidance is offered in the Commission's Guidelines on horizontal cooperation agreements [9]. The Guidelines suggest that competition restrictions can be avoided by carefully designing the way in which the shared costs of building and operating the network are allocated between participating operators. Restrictions can be avoided by allocating costs in a way that maintains the cost structure of individually deployed networks. In particular, the Commission warns that changing *fixed costs* – costs that do not vary with the number of subscribers – into *variable costs* – costs that depend on the number of subscribers – can disincentivize operators from competing in the service market.[3] For example, an NSA that allocates costs based on usage can disincentive an operator from attracting new subscribers. As mobile networks largely incur fixed costs [11], [12][4], the NSA cost structure should contain mostly fixed costs to meet the guidelines. More serious concerns can arise if the shared costs become a significant amount of the operators' total costs[5], a scenario the Commission refers to as a *significant commonality of costs*. In this scenario, the Guidelines warn that the sharing of variable costs can allow operators to more easily coordinate market prices and reach cartel-like price fixing outcomes, a conclusion which is corroborated by economic research [13]. This concern particularly applies to 5G, as 5G properties like dense node deployment, network sharing

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1. The Commission recently sent a Statement of Objections to T-Mobile, O2 and CETIN based on their 2G/3G/4G NSA in the Czech Republic [2]. See also the Commission's merger investigation into Telecom Italia and Vodafone [3].

2. During a recent evaluation [8] of the European Commission's Guidelines [9], some EU stakeholders reported feeling that the Guidelines encouraged companies to perform an overly cautious self-assessment of their cooperation agreements.

3. In 2012, Danish regulators investigated the Telia and Telenor NSA based in part on the variable cost structure of the agreement [10].

4. RAN costs are roughly proportional to the cost of RAN equipment and spectrum bandwidth [12]. Thus, assuming that the quantity of RAN equipment and spectrum is fixed in the long-term, RAN costs are largely fixed costs.

5. Total costs refers to the sum of the operators' individual costs and shared costs.

in urban areas, full-RAN/spectrum sharing, and partial core sharing (e.g., sharing edge computing resources) may result in a significant commonality of costs compared to previous mobile generations. Hence, MNOs should avoid creating variable costs when allocating the costs of shared 5G networks.

In spite of the Commission's guidelines and the cited concerns, research efforts to find economically viable NSAs have largely focused on cost structures that change fixed costs into variable costs [14]–[19]. These variable costs are the result of inter-MNO resource pricing mechanisms. Pricing is used to incentivize dynamic and efficient resource trading between operators (or between an operator and a 3rd party, e.g., joint venture) as operators' needs change over time.[6] While pricing allows efficient resource allocation, this article considers the resulting cost structures as idealized models which some operators may avoid in the future to guarantee regulatory compliance. Instead, we consider how operators can dynamically share resources under a fixed cost constraint, i.e., without pricing mechanisms. When pricing is not available, incentivizing sharing can be a challenging task. The details of this challenge are discussed in Section 2.

1.2 Contributions

In this article, we propose a novel network sharing framework which allows two MNOs to dynamically reallocate resources under a fixed cost constraint. To ensure that network sharing costs remain fixed, our framework avoids using a short/medium-term resource pricing mechanism. In our NSA, both MNOs pay fixed costs for an equal long-term share of the jointly owned network resources (i.e., joint venture). Our framework allows an MNO to share resources with another MNO as needed over a medium-term basis. We present our framework in Section 3.

Duopoly Market Model (Section 4 and Section 5) We evaluate our framework in a duopoly market model with an incumbent MNO and new entrant MNO. The new entrant decides what resources it shares with the incumbent in the medium term, depending on the revenue it expects to earn later after selling services to end-users. After the new entrant decides what resources to share, both MNOs independently choose the price an end-user must pay for subscription service. In turn, each end-user decides which MNO to subscribe to, depending on the MNOs' service prices and delivered quality-of-service (QoS). We show that the decisions of the MNOs and end-users reach a unique equilibrium.

How is (medium-term) sharing incentivized? (Section 5.3) At equilibrium, the new entrant shares exactly enough resources to maximize its profit. *We find that sharing can increase profit by allowing the new entrant to differentiate its services (in terms of service prices and delivered QoS) from the incumbent's services, and thus better compete with the incumbent for service revenue.* This finding is related to a known result on NSAs [20]: given the choice between a cost allocation that preserves fixed costs and a cost allocation that variably depends on number of subscribers, MNOs prefer to preserve fixed costs when their services are differentiated. Indeed, we find that MNOs prefer to differentiate services when their cost allocation rule preserves fixed costs.

When do MNOs share? (Section 5.3) We compare the amount of resources shared to several market and network factors. We find that the new entrant shares more resources when there are a fewer number of end-users in the market. In contrast, more resources are shared when the incumbent's network and new entrant's network exhibit *strong isolation* – a QoS specification of network slicing. *In fact, MNOs can offset the effect of market factors on the sharing equilibrium by controlling network factors.* Fine-tuned control of network factors is made possible by leveraging the network slicing capabilities of 5G networks.

How does sharing impact the end-user? (Section 5.3) We find that sharing has a mixed effect on the end-user's welfare. On the positive side, sharing benefits the end-user by maximizing delivered QoS. On the negative side, sharing can harm the end-user by increasing service prices. The net effect is a subscriber surplus of zero.

Is our framework viable from a regulatory/economic perspective? (Section 5.4) To assess the regulatory and economic viability of our framework, we compare our framework to several baseline scenarios. We find that when an NSA relaxes the fixed cost constraint, the MNOs may choose monopoly prices. This behavior is expected, as previously known results indicate that NSAs can result in anti-competitive pricing when MNOs share variable costs [13]. Furthermore, we identify that our NS framework meets certain regulatory objectives specified by the Body of European Regulators for Electronic Communications (BEREC) *conditional* on the MNO's choice of network parameters. Suggestions for choosing these parameters are discussed.

Finally, we assess the economic viability of our framework by evaluating MNOs' profits before and after adopting our framework. This evaluation suggests that as legacy networks are phased out, the opportunity cost of not adopting our framework will increase. *Hence, we anticipate that our framework will increase in profitability as the 5G rollout matures and 5G services become more distinguished from legacy network services.*

Does our framework generalize to other settings? (Section 6) We depart from the incumbent/new-entrant setting to evaluate our framework for network sharing between two *incumbent MNOs*. Simulations demonstrate that with 2 incumbents, our framework does not always incentivize and MNO to share.

2 Related Work

Ex-ante Resource Allocation: In the literature, a number of resource allocation mechanisms have been proposed which keep individual costs fixed [20]–[23]. We broadly refer to these mechanisms as ex-ante allocations as they split shared resources and costs *ex-ante* – before market shares (i.e., number of subscribers) are observed. For example, an ex-ante allocation may split resources and costs in proportion to an MNO's expected number of subscribers (e.g., budgeted output payments [20], risk-sharing investments [21]). Since ex-ante allocations do not depend on observed market shares, they do not violate the fixed cost constraint. However, an MNO may require more resources than it has been allocated if observed market shares differ significantly from expected market shares. It is not guaranteed that an MNO with a resource surplus will voluntarily reallocate some resources to an MNO with a resource deficit, as MNOs

6. Time is an important factor in the distinction between fixed and variable costs. Any cost may be regarded as variable if it can change within a short timescale.

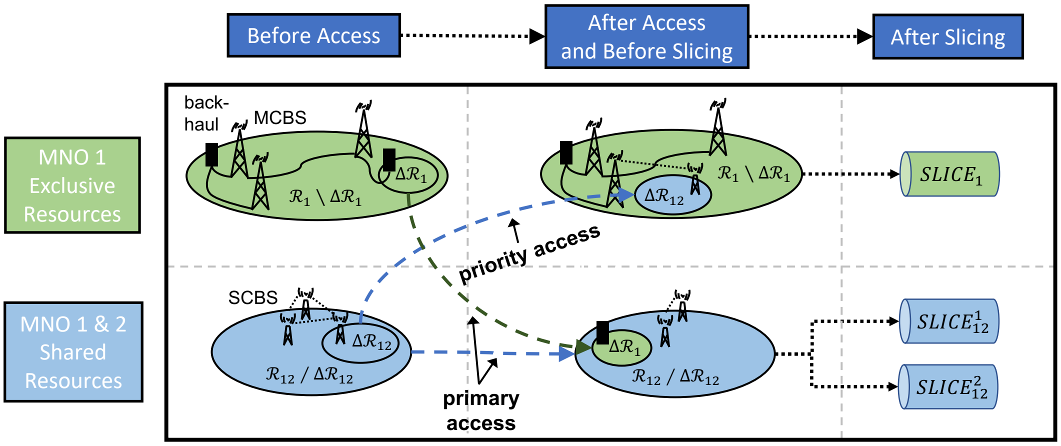

Fig. 1: Network sharing framework for MNO 1 (incumbent) and MNO 2 (new entrant).

are profit driven and costs cannot be reallocated from the latter to compensate the former. Therefore, solutions other than ex-ante allocations are needed to incentivize voluntary resource reallocation. This article addresses the problem of incentivizing MNOs to reallocate resources *ex-post* – after market shares are observed – without changing the ex-ante cost allocation.

Ex-post Resource Allocation: One potential solution to the above problem is multi-operator scheduling [24]–[29]. Scheduling allows MNOs to temporarily reallocate shared resources on a short-term basis. Here, short-term means that RAN conditions (e.g., load per MNO) may be variable while market shares are fixed. Resource pricing is not typically used to incentivize MNO participation in scheduling, and, therefore, fixed costs remain trivially fixed. Instead of pricing, the scheduling policy can incentivize participation by fairly allocating resources in a manner that benefits all MNOs. However, if an MNO's demand for resources changes due to a change in market share, scheduling cannot flexibly adapt without violating fairness. Hence, scheduling is not viable for resource reallocation in the *medium-term* – a time frame which is long enough for market shares to be variable but too short for costs to be reallocated. This article focuses on resource reallocation in the medium-term.

In addition to the scheduling literature, a few other works have explored concepts that may be applied to our problem. Revenue sharing between MNOs is one concept that can incentivize resource reallocation without cost reallocation [30]. Revenue sharing is likely to be seen by regulators as a clear restriction of competition. Opportunistic access of underutilized resources is another potential solution. This has been explored mostly in spectrum sharing scenarios involving primary (licensed) users and secondary (unlicensed) users [31], but also for resource sharing at the network level [32]. Opportunistic access can reduce underutilization of resources but does not allow flexible medium-term sharing.

Markets with NSAs: Our article fits into a larger body of literature which has studied the economic incentives for operators to participate in an NSA [15], [33]–[39]. In [15], [33], [34], [37]–[39], the authors consider NS in an existing network and allow participating operators to trade resources via resource pricing. These works study the impact of resource pricing on prices charged to the end-user. Some works additionally study the impact of resource pricing on user welfare including received quality-of-service (QoS) [15], [37]–[39] and geographic coverage [37]. More closely related to the present article, a few of the above works avoid resource pricing altogether [35], [36]. These works avoid pricing by using a cooperative game theory framework to allocate costs of the shared network between participating operators before the costs are accrued. We remark that the cost allocation rules considered in [35], [36] can depend on operators' market shares and thus violate our fixed cost constraint.

Similar to many of the above studies, we study the impact of resource sharing on the price charged by each operator to the end-user, as well as the impact on end-user welfare. In contrast to the above studies, we focus on operator incentives to share/trade resources when inter-MNO pricing is not available. Straightforward extensions of the above studies (e.g., fixing inter-MNO prices to 0) are not sufficient to address this focus and result in a trivial equilibrium where no resources are shared/traded. Thus, a new framework is needed to incentive sharing without pricing.

Beyond the ecomonic incentives of NSAs, some works have focused on the technological aspects of NSAs. Many works have focused on RAN sharing including vertical handoff [40], spectrum sharing [36], [38], [41], [42], Wifi offloading [43], and basestation sharing [37]. More broadly than RAN sharing, some works consider end-to-end *network slicing* [15], [38], [44] which the present article also considers. Network slicing allows two or more operators to dynamically share a common infrastructure via resource virtualization, and is enabled by 5G technologies like software defined networking (SDN) and network function virtualization (NFV). Slicing is used in [15], [38], [44] to enable resource trading between operators while satisfying end-user quality-of-service guarantees. In this article, we consider whether slicing can be leveraged to incentivize sharing in the absence of resource pricing.

3 Network Model and NS Framework

In this section, we introduce our network model and network sharing framework. We describe how resources shared under our framework are sliced according to network slice QoS specifications. Lastly, we introduce our model of subscriber utility and its relationship to the slice QoS specifications.

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3.1 Network Model

Consider a mobile network (depicted in Fig. 1) in a fixed geographical area operated by an incumbent MNO and new entrant MNO, referred to as MNO 1 and MNO 2, respectively. From the perspective of the mobile users, MNO 1 and MNO 2 are competing service providers, e.g., selling subscriptions for enhanced mobile broadband (eMBB) or ultra-reliable and low-latency communications (uRLLC) services in the area. Examples of such services include VR, real-time AI, and vehicular applications. Note that these services may have stringent QoS requirements. Although we focus on 5G NSAs due to their topicality, we present our framework in a general way that may be applicable for future network deployments.

User pool: We model the pool of mobile users as a continuum where the fraction of the pool that purchases a service subscription from MNO 1 and MNO 2 are $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$, respectively, where $\lambda_1 + \lambda_2 \leq 1$. The fraction of the pool that opts out of service is $1 - \lambda_1 - \lambda_2$. Often, we refer to λ_i as the *market share* of MNO i. Let $\Lambda > 0$ be the total number of users (i.e., the mass of the continuum), where it follows that $\Lambda\lambda_i$ is the number of users subscribed to MNO i for $i = 1,2$. In general, each user may have a different QoS requirement for which its service must be tailored accordingly. To accommodate heterogeneous QoS needs, MNO i may create multiple slices through its network, each tailored for a different service level agreement (SLA) such that one slice serves all subscribers with similar QoS needs. Therefore, we may assume that all users served on the same slice have homogeneous QoS needs. In the sequel, we restrict our attention to one slice of MNO i's network, and discuss in Appendix A the extension to the multi-slice case with heterogeneous QoS requirements. In the sequel, we redefine the user pool to be all users requesting the same SLA, i.e., all Λ users have homogeneous QoS requirements.

Infrastructure and costs: The mobile network infrastructure in the area consists of a collection of networks resources. A set of resources $\mathcal{R}_1$ is owned only by MNO 1 as an individual legacy network deployment. The set $\mathcal{R}_1$ consists of a backhaul network as well as some legacy network resources, e.g., legacy 3G/4G resources in the radio access network (RAN) (e.g., macrocell base stations (MCBSs)). To build and operate the resources $\mathcal{R}_1$, MNO 1 pays a *fixed cost* $q_1 > 0$ where fixed means that q_1 does not depend on $\lambda_1\Lambda$ and $\lambda_2\Lambda$.

To build a 5G/beyond-5G network deployment, the MNOs form a joint venture and cooperatively deploy a single network that both MNOs share. By forming a joint venture, the MNOs can reduce infrastructure costs compared to the scenario where both MNOs deploy separate networks. We denote the set of resources owned by the joint venture as $\mathcal{R}_{12}$. The set $\mathcal{R}_{12}$ consists of local infrastructure, e.g., dense network of small cell base stations (SCBSs), edge computing resources. Since MNO 2 is a new entrant, we assume that MNO 2 *does not* own individually deployed network resources. Together, MNO 1 and MNO 2 pay a fixed cost $q_{12} > 0$ for the resources $\mathcal{R}_{12}$. This cost is split between MNO 1 and MNO 2 by a cost transfer $\Delta q_{12} \in [0, q_{12}]$ such that MNO 1 pays for the cost Δq_{12} and MNO 2 pays for the cost $q_{12} - \Delta q_{12}$. In total, for $i = 1,2$, MNO i pays a total cost q_i^{total} such that

$$q_1^{total} = q_1 + \Delta q_{12}, \qquad q_2^{total} = q_{12} - \Delta q_{12}. \tag{1}$$

We remark that our fixed cost constraint implies that the transfer Δq_{12} is a fixed cost, i.e., Δq_{12} does not depend on $\lambda_1\Lambda$ and $\lambda_2\Lambda$, and thus q_1^{total} and q_2^{total} remain fixed costs.

3.2 Network Sharing Framework

Our network sharing framework allows MNOs to dynamically share the jointly owned resources $\mathcal{R}_{12}$ in the *medium-term*, i.e., a time frame where λ_1 and λ_2 are variable. To decide how $\mathcal{R}_{12}$ is shared in this time frame, the MNOs select two network specifications: the resource *access policy* and the *slice QoS specifications*.

Access Policy: The MNOs choose which MNO has access to resources in the set $\mathcal{R}_{12}$ by selecting the access policy. For a given resource in a resource set, a resource's access policy dictates how each MNO's subscribers can access that resource. We consider 2 policies: *primary access* and *priority access*. Under a primary access policy, a resource services the traffic of all subscribers with equal priority regardless of their home MNO. Under a priority access policy, a resource only serves the traffic of a given MNO's subscribers. In the long-term, we assume that both MNOs agree to equally share the jointly owned resources by assigning a primary access policy to all resources in $\mathcal{R}_{12}$. Since no resource in $\mathcal{R}_1$ is shared with MNO 2, we say that MNO 1 has priority access to all of $\mathcal{R}_1$.

Slice QoS Specifications: After the MNOs select the access policy, resources $\mathcal{R}_{12}$ are sliced. Network slicing allows each MNO to build a logical end-to-end network from $\mathcal{R}_{12}$ under certain performance guarantees. These performance guarantees are defined by the MNOs via the *slice QoS specifications*. In Section 3.3, we detail how MNOs can control these specifications via the slice overbooking policy.

Medium-term sharing: Together, the access policy and slice QoS specifications offer the MNOs some degrees-of-freedom to share resources in the medium-term without varying the cost transfer Δq_{12}. This sharing can allow MNOs to respond to changes in market conditions. For example, if MNO 1 sees an increase in its number of subscribers $\lambda_1\Lambda$ and correspondingly requires additional resources, this can be achieved by modifying the access policy of a subset of resources in $\mathcal{R}_{12}$ from primary access to priority access (for MNO 1).

Access Conditions: We now consider certain conditions on how the MNOs can modify a resource's access policy. To incentivize voluntary participation in network sharing, an access policy must be modified in a way that is fair to both MNOs. Furthermore, the MNOs must be able to modify an access policy in a way that does not enable collusion. We summarize our conditions for modifying a resource's access policy in the following three Conditions.

1) (Default Access) The default access policy for all resources in $\mathcal{R}_{12}$ is primary access.
2) (Fair Access) If an access policy is modified, then the modified policy must satisfy the following fairness condition: For $i, j = 1,2$ and $i \neq j$, a policy cannot be modified to give MNO i priority access to a given resource *unless* MNO j agrees to the new policy.
3) (Non-cooperative Access) A resource's access policy is modified non-cooperatively by the new entrant MNO 2.

Condition 1 is consistent with the MNOs' long-term agreement to equally share the jointly owned resources. Condition 2 states that the access policy cannot be modified in a way that violates an MNO's right to access the shared resources. In some cases, an MNO may temporarily waive this right over a given resource. Lastly, Condition 3 safeguards against MNO collusion by requiring that the modified access policy be chosen non-cooperatively, i.e., unilaterally by MNO 2. Furthermore, Condition 3 minimizes the incumbent MNO 1's control over the access policy. This may help to avoid increasing the dominance of an MNO that may already have substantial market power.

We now introduce our network sharing framework which is consistent with the above three Conditions. Our framework is illustrated in Fig. 1 at three different stages of network sharing: before access, after access and before slicing, and after slicing. To satisfy Condition 3, the access policy of $\mathcal{R}_{12}$ is set by the operator of the jointly owned local network, i.e., the new entrant MNO 2. To grant priority access, MNO 2 selects a subset $\Delta\mathcal{R}_{12}$ from $\mathcal{R}_{12}$ and gives MNO 1 priority access to each resource in $\Delta\mathcal{R}_{12}$. The remaining resources $\mathcal{R}_{12} \setminus \Delta\mathcal{R}_{12}$ retain the default primary access policy. To satisfy Condition 2, we do not allow MNO 2 to grant itself priority access to any subset of $\mathcal{R}_{12}$.

Lastly, we consider the possibility that MNO 1 may share some of its own resources $\mathcal{R}_1$ with MNO 2. Regulators may require that the incumbent provide the new entrant essential connectivity to outside the network so that MNO 2 is capable of delivering basic services to its subscribers [45]. Hence, we assume that MNO 1 provides these essential connectivity resources to MNO 2. Beyond these essential resources, we let MNO 1 select a subset $\Delta\mathcal{R}_1$ of $\mathcal{R}_1$ and assign primary access to $\Delta\mathcal{R}_1$, as illustrated in Fig. 1.

3.3 Network Slicing

Slicing begins with two inputs provided by the MNOs: the access policy and slice QoS specifications. As the output of network slicing, each MNO is delivered one slice. As illustrated in Fig. 1, an MNO's slice is a composition of two subslices where one subslice is orchestrated from reserved resources while the other is orchestrated from shared resources. In particular, MNO 1 is delivered a subslice $SLICE_1$ orchestrated from resources with a priority access policy: $\mathcal{R}_1 \setminus \Delta\mathcal{R}_1$ and $\Delta\mathcal{R}_{12}$, and another subslice $SLICE_{12}^1$ orchestrated from resources with a primary access policy: $\mathcal{R}_{12} \setminus \Delta\mathcal{R}_{12}$ and $\Delta\mathcal{R}_1$. Similarly, MNO 2 is delivered a subslice $SLICE_{12}^2$ orchestrated from resources with a primary access policy: $\mathcal{R}_{12} \setminus \Delta\mathcal{R}_{12}$ and $\Delta\mathcal{R}_1$.

All subslices are orchestrated according to slice QoS specifications negotiated between the MNOs. Following the modeling work of [46], we consider two slice QoS specifications: *guaranteed demand* and *overbooking penalty*. These specifications play a leading role in characterizing subscribers' received QoS.

3.3.1 Guaranteed Demand

Slicing is performed to provide QoS guarantees for the users' traffic it serves. Realistically, a slice (or subslice) may not be able to provide QoS guarantees for all serviced traffic. Some traffic may receive QoS violations. Thus, we consider a subslice's guaranteed demand, defined as the percentage of all traffic serviced by a subslice according to the users' requested QoS. We assume that any traffic outside of this percentage is provided with best-effort service that may violate the requested QoS. Let the guaranteed demand of subslices $SLICE_1$ and $SLICE_{12}^i$ be $s_1 \in [0,1]$ and $s_{12}^i \in [0,1]$, respectively. On a user-by-user case, we make the following assumption: For any users with services supported by subslice $SLICE_1$ ($SLICE_{12}^i$), the event that a user is guaranteed to receive their requested QoS is i.i.d. with probability s_1 (s_{12}^i). Otherwise, a user receives a QoS violation. In practice, a user's empirical probability of receiving its requested QoS may be lower due to resource overbooking. Overbooking is discussed in Section 3.3.2.

Guaranteed demand is closely related to the quantity of a subslice's underlying resources. Intuitively, the more (less) resources available to a subslice, the larger (smaller) its guaranteed demand. We denote the maximum guaranteed demand supported by the resource sets $\mathcal{R}_1$, $\mathcal{R}_{12}$, $\Delta\mathcal{R}_1$, and $\Delta\mathcal{R}_{12}$ as the values (in [0,1]) d_1, d_{12}, Δd_1, and Δd_{12}, respectively. Without loss of generality, we normalize the above values such that their sum is less than or equal to 1. With some accounting, we find that the guaranteed demand of subslice $SLICE_1$ is equal to $s_1 = d_1 + \Delta d_{12} - \Delta d_1$. Since both MNOs are given primary access to the set $\mathcal{R}_{12} \setminus \Delta\mathcal{R}_{12}$, these shared resources are fairly sliced such that both $SLICE_{12}^1$ and $SLICE_{12}^2$ are specified with equivalent guaranteed demand. Hence, the guaranteed demand of $SLICE_{12}^i$ is $s_{12}^i = s_{12} = d_{12} - \Delta d_{12} + \Delta d_1$ for $i = 1,2$.

3.3.2 Overbooking Penalty

In addition to guaranteed demand, we consider slice QoS specifications related to resource overbooking. In the context of slicing, overbooking occurs when the same virtualized resource is allocated to two or more subslices. In [46], resource overbooking has been identified as a promising strategy for improving network utilization during network slicing.

When an overbooked subslice requires all booked capacity, congestion will occur in the overbooked resources. As a result, some percentage g of the guaranteed demand will not receive the quality-of-service (QoS) promised in the SLA, but instead, be treated according to best-effort service. Hence, overbooking can result in QoS violations. Intuitively, the more traffic that is serviced by overbooked resources, the larger g will be. Therefore, we let g be an increasing function of the number of users sharing the overbooked resources. Recall that for $i = 1,2$, the number of users sharing overbooked resources to subslice $SLICE_1$ and $SLICE_{12}^i$ is $\lambda_1\Lambda$ and $(\lambda_1 + \lambda_2)\Lambda$, respectively. We assume that event that a user receives a QoS violation due to overbooking is i.i.d. with probability $g(\cdot)$. Therefore, a user's empirical probability of receiving its requested QoS is the guaranteed demand multiplied with $(1 - g(\cdot))$. We refer to this probability as the *subslice QoS*, where the subslice QoS of of $SLICE_1$ and $SLICE_{12}^i$ is $QoS_1 = s_1 - g(\lambda_1\Lambda)s_1$ and $QoS_{12} = s_{12} - g((\lambda_1 + \lambda_2)\Lambda)s_{12}$, respectively.

3.4 Services

The utility that users derive from services is affected by subslice QoS and pricing in the service market. To motivate our user utility model, we consider several properties that impact a user's service experience.

3.4.1 Subscription

To subscribe to MNO $i = 1,2$, a user must pay the MNO a service price $p_i > 0$ set by the operator. Once set, this subscription fee is fixed for the medium-term.

3.4.2 Service valuation

The subscriber receives a service valuation that depends on how its MNO handles its traffic. If the subscriber's traffic is serviced according to the requested QoS, the subscriber receives a positive valuation of v. Otherwise, if the subscriber's traffic is given best-effort service, then the subscriber receives a valuation of 0. Since the user pool is pre-selected to have similar QoS requirements, we consider service valuation v to be homogeneous. Therefore, the expected valuation for a subscriber to MNO 1 and MNO 2 is $v(QoS_1 + QoS_{12})$ and $v(QoS_{12})$, respectively.

3.4.3 Linear Congestion

When overbooking occurs, overbooked resources are allocated to users according to an overbooking scheduling policy. The scheduling policy determines when users are denied access to resources, and, hence, we may consider the congestion function $g(\cdot)$ to be a property of the scheduling policy. Although we do not assume a particular policy, we restrict g to be linear in the number of users using overbooked resources, i.e., for a constant $\alpha > 0$, $g(x) = \alpha x$ if $\alpha x \leq 1$, $g(x) = 1$ otherwise. A linear g may correspond to a scheduling policy that prioritizes fair resource allocation to all users. For example, in [47], the use of a round-robin scheduling policy to schedule traffic in a small cell network has an observed delay outage probability (i.e., congestion) that is approximately linear when traffic is heavy. The parameter α depends on the MNOs' overbooking policy where large (small) α corresponds to aggressive (conservative) overbooking. Note that by construction, $QoS_i + QoS_{12} \leq 1$, $QoS_i \geq 0$, and $QoS_{12} \geq 0$. Putting the above properties together, it follows that the subslice QoS terms are $QoS_1 = s_1 - \alpha s_1 \lambda_1 \Lambda$ and

$$QoS_{12} = s_{12} - \alpha s_{12}(\lambda_1 + \lambda_2)\Lambda, \tag{2}$$

and the expected utility of a subscriber to MNO 1 is

$$\begin{aligned} u_1(\lambda_1, \lambda_2) &= v(QoS_1 + QoS_{12}) - p_1 \\ &= v(s_1 + s_{12}) - \beta s_1 \lambda_1 - \beta s_{12}(\lambda_1 + \lambda_2) - p_1 \end{aligned} \tag{3}$$

and

$$\begin{aligned} u_2(\lambda_1, \lambda_2) &= v(QoS_{12}) - p_2 \\ &= v s_{12} - \beta s_{12}(\lambda_1 + \lambda_2) - p_2 \end{aligned} \tag{4}$$

where $\beta = v\alpha\Lambda$. For brevity, we simply refer to (3) and (4) as the user utility. In the context of discussing user welfare, we sometimes refer to (3) and (4) as user surplus.

4 Market Model

We turn our attention to the service market depicted in Fig. 2, which we model as a three-stage game played by MNO 1, MNO 2, and the mobile user pool. To satisfy the fixed cost constraint, the cost transfer Δq_{12} is set to a constant value prior to the game that cannot vary throughout the game. Thus, Δq_{12} is exogenous to our model.

4.1 Stage 1: Priority Access

In the first stage, the MNOs choose their resource access policies. Note that the user utilities (cf. (3)) only depend on the access policies in terms of guaranteed demand. Consequently, a market equilibrium can only depend on guaranteed demand. Therefore, without loss of generality, we let MNO 1 and MNO 2 choose Δd_1 and Δd_{12} as their first stage strategies, respectively. To reflect this choice, throughout the rest of the article we refer to Δd_1 and Δd_{12} as the *primary access strategy* of MNO 1 and *priority access strategy* of MNO 2, respectively. Neither operator can give access to more resources than are available, and therefore, the strategies are bounded by $0 \leq \Delta d_1 \leq d_1$ and $0 \leq \Delta d_{12} \leq d_{12}$. MNO 1 and MNO 2 choose their access policies to maximize profit $\pi_1 = p_1\lambda_1\Lambda - q_1^{total}$ and $\pi_2 = p_2\lambda_2\Lambda - q_2^{total}$, respectively.

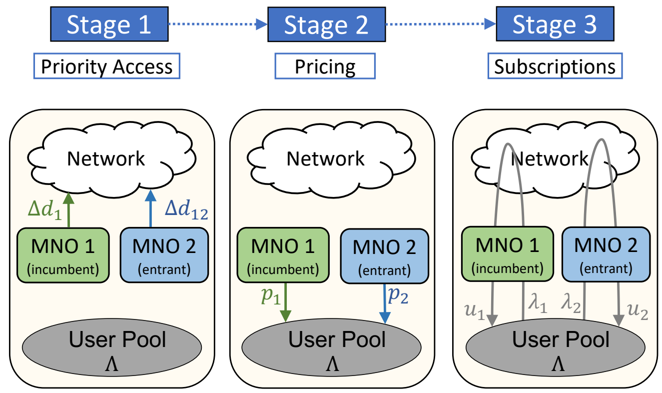

Fig. 2: Service market as a three-stage game.

4.2 Stage 2: Service Pricing

Once the MNOs have chosen their access strategy, each MNO may set service prices. For $i = 1, 2$, MNO i chooses a service price $p_i \geq 0$. We assume that the price p_i is a flat-rate service price. Currently, mobile services are charged at a monthly flat-rate; thus, our assumption is in accordance with the current practice. MNO i chooses its service price to maximize profit π_i.

4.3 Stage 3: Mobile User Subscriptions

In the third stage, the pool of mobile users selects subscriptions in an one shot game. To maximize its utility, each user may subscribe to either one MNO or forgo service with no subscription. If a user subscribes to MNO 1 or MNO 2, its utility is given by (3) or (4), respectively. If a user chooses not to subscribe to either operator, its utility is 0. To streamline our equilibrium analysis, we assume that a user is able to know its utility ahead of choosing a subscription. However, such an assumption is not necessary for users to learn the equilibrium if we were to extend this one shot game to a repeated game. In the following subsection, we discuss how best response dynamics would allow the users to converge to the equilibrium without knowing their utilities.

4.4 Multi-Stage Market Equilibrium

Assuming that all players will make rational decisions in future stages, we proceed with equilibrium analysis using backward induction. We analyze the equilibrium of each stage separately, beginning with the third stage while assuming the strategies of the previous stages are fixed. In the third stage, we seek to characterize the *Wardrop equilibrium* (Definition 1), which can be shown to be a Nash Equilibrium. In the second and first stages, we characterize the Nash Equilibrium. We refer to a sub-game perfect Nash equilibrium of this multi-stage game as a *market equilibrium*.

We now comment on a possible generalization of the third-stage Wardrop equilibrium. In Section 4.3 we assumed that a user is able to know its utility ahead of choosing a subscription. Assume instead that users do not know their utility expression (3), but can only observe an evaluation of their utility after choosing a subscription. Here, we redefine the third-stage so that users can play a *dynamic game*, interatively updating their strategies over a discrete time horizon. We argue that the users' subscription choices in the dynamic game converge to an equilibrium of the one shot game (Section 4.3) where users know their utilities.

Consider a modified user pool model where the number of users is a large, but finite, number M. Let $\boldsymbol{a}$ denote the users' subscription profile where the jth element a_j denotes the jth user's subscription for $j = 1, \dots, M$. The M users update their subscriptions in the following manner: at the current time step, pick an arbitrary j and let the jth user choose an updated subscription a'_j to maximize its utility $u^j(a'_j, \boldsymbol{a}_{-j})$.[7] It is easy to show that the users' utilities admit an *exact potential function*, that is, there exists a function $P(a_j, \boldsymbol{a}_{-j})$ such that for any user j and any a'_j, a''_j, and $\boldsymbol{a}_{-j}$, we have that $u^j(a'_j, \boldsymbol{a}_{-j}) - u^j(a''_j, \boldsymbol{a}_{-j}) = P(a'_j, \boldsymbol{a}_{-j}) - P(a''_j, \boldsymbol{a}_{-j})$. Following the existence of the exact potential function, the game is guaranteed to converge to a Nash equilibrium [48, Theorem 19.12]. Since the Wardrop equilibrium is the Nash equilibrium, we conclude that generality is not lost by assuming that users know their utilities and use this information to compute the Wardrop equilibrium in a one shot game. We proceed by using the simpler Wardrop equilibrium conditions.

5 Market Equilibrium under NS Framework

In this section, we characterize the market equilibrium when MNOs can reallocate network resources via our NS framework. The third stage equilibrium (Section 5.1) and second stage equilibrium (Section 5.2) have already been characterized in [41] under a spectrum sharing model. We restate these results for the reader's convenience, accompanied by a new discussion focused around network sharing. We present our main findings in Section 5.3 and Section 5.4. Proceeding with equilibrium analysis, we apply backward induction and begin with the third stage game.

5.1 User equilibrium

We first characterize the third stage equilibrium, alternatively referred to as the user equilibrium. We define the user equilibrium in terms of the Wardrop equilibrium [49], which is a popular solution concept for network games with non-atomic[8] users [34], [41], [50].

Definition 1 (Wardrop Conditions)**.** *The profile (λ_1, λ_2) is the third-stage Wardrop equilibrium if the following two conditions are satisfied.*

1) (Substitute Services) If both MNOs have subscribers, i.e., $\lambda_1, \lambda_2 > 0$, then the utilities of all subscribers must be equal, i.e., $u_1(\lambda_1, \lambda_2) = u_2(\lambda_1, \lambda_2)$. If instead MNO i has subscribers and MNO j does not, i.e., $\lambda_i > 0$ and $\lambda_j = 0$, then the utility of a subscriber to MNO i must be larger than the utility of a subscriber to MNO j, i.e., $u_i(\lambda_1, \lambda_2) \geq u_j(\lambda_1, \lambda_2)$.

2) (Individual Rationality) For MNO $i = 1, 2$, if $\lambda_i > 0$, then $u_i(\lambda_1, \lambda_2) \geq 0$. Otherwise, if $\lambda_i = 0$, then $u_i(\lambda_1, \lambda_2) \leq 0$. Furthermore, if $\lambda_1, \lambda_2 > 0$ and $\lambda_1 + \lambda_2 < 1$, then $u_i(\lambda_1, \lambda_2) = 0$ for $i = 1, 2$.

The first condition states that a subscription to MNO 1 and MNO 2 are substitutes, i.e., a user will subscribe to the operator whose service will offer them the largest utility. Note that at equilibrium, both MNOs cannot have subscribers if u_1 and u_2 are not equal. If this scenario occurred, then subscribers with the lower utility would change their subscription to the service with a higher utility.

The second condition states that, in equilibrium, a user will not subscribe to an MNO if it expects to receive a negative utility. If a user anticipates its utility to be negative, it can always forgo service to receive a utility of 0. The second condition can also be considered as a special case of the first, i.e., we can consider service subscription and service opt out as substitutes. What follows is that if some users have chosen to subscribe to an MNO while others have opted out of service, then subscribing to MNO 1, subscribing to MNO 2, and service opt out must provide the same utility.

5.2 Price Equilibrium

We now turn to the second stage game where each MNO chooses their service prices. We refer to the equilibrium of the second stage game as the price equilibrium and denote it by the tuple (p_1, p_2). Define $\gamma = \nu/\beta = 1/(\alpha\Lambda)$ as the slice isolation factor. Recall that α is related to the MNOs' overbooking policy, where large (small) α corresponds to aggressive (conservative) overbooking. As overbooking increases the amount of resources shared between two slices, the resource isolation between these slices necessarily decreases. Therefore, γ describes the degree to which resources are isolated between slices, where large (small) γ corresponds to strong (weak) network slice isolation. We begin by characterizing the price equilibrium when γ is large.

Lemma 1. *If $\gamma \geq 2$, then only MNO 1 offers service and the price equilibrium is $p_1 = s_1(\nu - \beta)$. Furthermore, the market shares are $\lambda_1 = 1$ and $\lambda_2 = 0$, and the MNO profits are $\pi_1 = p_1\Lambda - q_1^{total}$ and $\pi_2 = -q_2^{total}$.*

We follow the convention that if an MNO has no subscribers, then the operator's price of the subscription is 0. Therefore, we set $p_2 = 0$ when $\gamma \geq 2$. The above result shows that if γ is large, MNO 2 may not have any customers. Therefore, MNO 2 will not enter the market, resulting in a monopoly for MNO 1. Intuitively, if γ is large, a user's expected payoff increases when they subscribe to MNO 1 compared to MNO 2. Hence, a higher value of γ will keep MNO 2 out of the market. The next equilibrium illustrates that for $\gamma < 2$, MNO 2 participates in the market.

Lemma 2 ([41, Theorem 5])**.** *If $\frac{2s_1+3s_{12}}{s_1+3s_{12}} < \gamma < 2$, then the unique price equilibrium is $p_1 = \frac{s_1}{3}(\nu + \beta)$ and $p_2 = \frac{s_1}{3}(2\beta - \nu)$.*

7. $u^j(a_j, \boldsymbol{a}_{-j})$ is defined as the utility (3) with λ_i redefined as $(1/M)$ times the number of users in the set $\{\text{user } k : a_k \textit{ indicates a subscription to MNO } i\}$.

8. In the context of our model, non-atomic means that an individual user's strategy does not affect the strategy of any other user.

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Furthermore, the market shares are $\lambda_1 = \frac{\nu+\beta}{3\beta}$ *and* $\lambda_2 = \frac{2\beta-\nu}{3\beta}$ *and the MNO profits are* $\pi_1 = \frac{s_1}{9\beta}(\beta+\nu)^2\Lambda - q_1^{total}$ *and* $\pi_2 = \frac{s_1}{9\beta}(2\beta-\nu)^2\Lambda - q_2^{total}$.

Similar to the previous equilibrium, no user here is unsubscribed, i.e., $\lambda_1+\lambda_2 = 1$. It is easy to verify that user surplus is strictly positive, i.e., $u_1 = u_2 > 0$, when the LHS of Lemma 2's condition holds with strict inequality. Note that service prices and MNO profits depend on s_1. As s_1 increases, the profits of both operators increase. Additionally, as MNO 2's priority access strategy Δd_{12} increases, s_1 increases. This suggests that priority access may be an effective strategy for increasing profit. Note that such an increase in Δd_{12} must also decrease s_{12}. *Since profits do not depend on* s_{12}*, this suggests that MNO 2 may be able to increase its profit at the expense of reducing the guaranteed demand of its slice.* Although counterintuitive, we explain in the following subsection why this strategy is effective and show when it occurs at market equilibrium. The next lemma characterizes the equilibrium when γ decreases to a value slightly above 1.

Lemma 3 ([41, Theorem 6])**.** *If* $\frac{4s_1+3s_{12}}{3(s_1+s_{12})} \le \gamma \le \frac{2s_1+3s_{12}}{s_1+3s_{12}}$*, then the unique price equilibrium is* $p_1 = s_1\frac{\nu}{2} + s_{12}\frac{\nu-\beta}{2}$ *and* $p_2 = s_{12}(\nu-\beta)$*. Furthermore, the market shares are* $\lambda_1 = \frac{s_1\nu+s_{12}(\nu-\beta)}{2\beta s_1}$ *and* $\lambda_2 = \frac{s_1(2\beta-\nu)-s_{12}(\nu-\beta)}{2\beta s_1}$ *and the MNO profits are* $\pi_1 = \left(s_1\frac{\nu}{2}+s_{12}\frac{\nu-\beta}{2}\right)^2\frac{\Lambda}{\beta s_1} - q_1^{total}$ *and* $\pi_2 = s_{12}(\nu-\beta)\frac{s_1(2\beta-\nu)-s_{12}(\nu-\beta)}{2\beta s_1}\Lambda - q_2^{total}$.

At this equilibrium, $\lambda_1+\lambda_2 = 1$, but user surplus is zero. Unlike the previous equilibrium, it is not immediately clear here if both operators receive higher profits by increasing access, as profits and the market shares of both operators are a function of s_1 and s_{12}. As with the previous equilibrium, the market share of MNO 1 and MNO 2 decreases and increases, respectively, as γ decreases. Intuitively, decreasing γ corresponds to decreasing service valuation ν (w.r.t. β), and hence, users are more willing to tolerate MNO 2's lower QoS in exchange for the lower service prices, as compared to MNO 1's service. Note that for a fixed γ, it is possible (depending on s_1 and s_{12}) for both MNOs to have larger profits compared to the previous equilibrium. This suggests that priority access may be an effective strategy for increasing profit when γ is slightly above 1. The next lemma characterizes the equilibrium when γ is small.

Lemma 4 ([41, Theorem 7])**.** *If* $\gamma < \frac{4s_1+3s_{12}}{3(s_1+s_{12})}$*, then the unique price equilibrium is* $p_1 = \frac{2\nu s_1(s_1+s_{12})}{4s_1+3s_{12}}$ *and* $p_2 = \frac{\nu s_1 s_{12}}{4s_1+3s_{12}}$*. Furthermore, the market shares are* $\lambda_1 = \frac{2\nu(s_1+s_{12})}{\beta(4s_1+3s_{12})}$ *and* $\lambda_2 = \frac{\nu(s_1+s_{12})}{\beta(4s_1+3s_{12})}$ *and the MNO profits are* $\pi_1 = \frac{s_1}{\beta}\left(\frac{2\nu(s_1+s_{12})}{4s_1+3s_{12}}\right)^2\Lambda - q_1^{total}$ *and* $\pi_2 = \frac{s_1 s_{12}(s_1+s_{12})}{\beta}\left(\frac{\nu}{4s_1+3s_{12}}\right)^2\Lambda - q_2^{total}$.

Unlike each of the previous equilibria, in this equilibrium, some users opt out of service by not subscribing to either MNO. This shows that when network slices are weakly isolated, i.e., when γ is smaller than some value slightly greater than 1, users experience heavy network congestion and the resulting low slice QoS discourages some users from participating in the market. Similar to the equilibrium of Lemma 3, it is not obvious by inspection whether priority access can increase MNO profit here. Therefore, to provide such insights, it is necessary to characterize the first stage equilibrium.

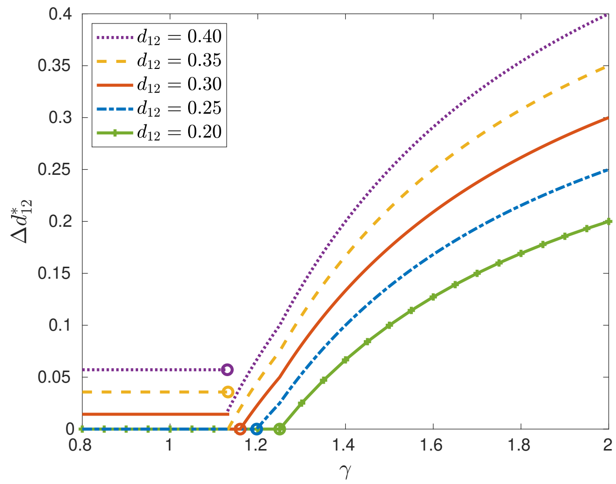

Fig. 3: First stage equilibrium strategy Δd_{12}^* where $d_1 = 0.2$.

5.3 Access Policy Equilibrium

We refer to the equilibrium of the first stage as the access equilibrium and denote it by the tuple $(\Delta d_1^*, \Delta d_{12}^*)$. Recall that $\gamma = \nu/\beta = 1/(\alpha\Lambda)$ is the slice isolation factor. We first present a result on MNO 1's priority access strategy.

Lemma 5 (no primary access)**.** *For any value of* $\gamma > 0$*, there does not exist an equilibrium path where MNO 1 will share resources with primary access, i.e.,* $\Delta d_1^* = 0$.

The above lemma follows from the observation that $\partial\pi_1/\partial\Delta d_1 < 0$ for any value of Δd_1 and γ, implying that MNO 1's second stage equilibrium profit is uniquely maximized at $\Delta d_1 = 0$. As a result, MNO 1 will not give primary access of any resources to MNO 2. The above lemma supports a conclusion of [45] which states that regulatory access is needed to provide MNO 2's subscribers end-to-end connectivity.

We now focus our attention on MNO 2's priority access equilibrium strategy Δd_{12}^*. Due to its lengthy presentation, the exact characterization of Δd_{12}^* is deposited in Appendix B. As a summary of this characterization, the following Theorem is structured to highlight Δd_{12}^*'s dependency on the degree of network slice isolation.

Theorem 1 (access equilibrium)**.** *For some number* $\Gamma(d_1, d_{12}) > 0$*, the unique first stage equilibrium is given by* $\Delta d_1^* = 0$ *and*

$$\Delta d_{12}^* = \begin{cases} d_{12}, & \gamma \ge 2 \\ h_1(d_1, d_{12}, \gamma), & \Gamma(d_1, d_{12}) < \gamma < 2 \\ h_2(d_1, d_{12}, \gamma), & \gamma \le \Gamma(d_1, d_{12}) \end{cases} \quad (5)$$

where h_1 *and* h_2 *are functions of* d_1*,* d_{12}*, and* γ *with the following properties:*

1) $h_1(\cdot, d_{12}, \cdot)$ *and* $h_2(\cdot, d_{12}, \cdot)$ *are strictly less than* d_{12}.
2) $h_1(\cdot, \cdot, \gamma)$ *is positive and an increasing function of* γ.
3) *For* $d_{12} \le \frac{4}{3}d_1$*,* $h_2 \equiv 0$.
4) *There exists a constant* $c > \frac{4}{3}$ *such that for* $d_{12} \ge c d_1$*,* $h_2(d_1, d_{12}, \gamma)$ *is positive and independent of* γ.

Interpretation of Access Equilibrium: Fig. 3 illustrates the above Theorem, showing Δd_{12}^* plotted against γ where circular line markers indicate $\Gamma(d_1, d_{12})$. The value Γ can be interpreted as a threshold between strong and weak slice isolation. When the network slices are strongly isolated, (i.e., $\gamma \ge \Gamma$), MNO 2

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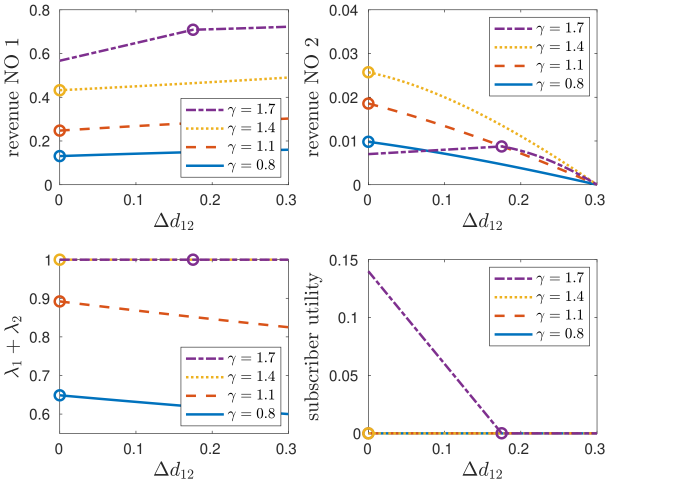

Fig. 4: Second stage price equilibrium as a function of Δd_{12} for $d_1 = 0.7$ and $d_{12} = 0.3$. Circular line markers indicate the first-stage/priority-access equilibrium.

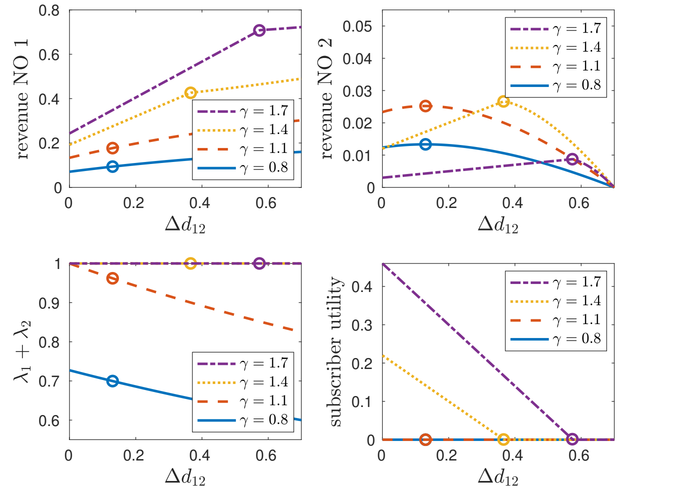

Fig. 5: Second stage price equilibrium as a function of Δd_{12} for $d_1 = 0.3$ and $d_{12} = 0.7$. Circular line markers indicate the first-stage/priority-access equilibrium.

always offers some priority access to MNO 1. The stronger the isolation, the more resources MNO 2 offers to MNO 1 with priority access. This trend continues until γ increases to 2 and MNO 2 gives MNO 1 priority access to all of its network resources. Here, MNO 2 has no resources to support subscriber services, and therefore, leaves the market (as consistent with Lemma 1).

NS decisions: Theorem 1 provides insight into how MNOs can share resources in the medium-term under a fixed cost constraint. MNOs can control the amount of resources they share by choosing their overbooking parameter α. To share more resource, MNOs should choose a more *conservative* overbooking policy (i.e., smaller α). To share fewer resources, MNOs should choose a more *aggressive* overbooking policy (i.e., larger α). Theorem 1 also shows that the amount of shared resources is sensitive to certain *market conditions*, in particular, the total number of users Λ. This follows from the observation that $\gamma = \frac{1}{\alpha\Lambda}$ is a function of Λ. *However, MNOs can offset the impact of this market factor on resource sharing by choosing an appropriate overbooking policy.*

To understand how sharing impacts other parts of the market, we turn our attention to the unique *market equilibrium* which occurs when MNOs choose access $(\Delta d_1, \Delta d_{12})$ according to the access equilibrium, MNOs choose service prices (p_1, p_2) according to the pricing equilibrium, and users choose their subscription numbers (λ_1, λ_2) according to the user equilibrium.[9]. The market equilibrium is illustrated in Fig. 4 and Fig. 5 which show MNO profit, total number of subscribers, and subscriber utility. Here, profit and utility are normalized by $\beta\Lambda$ and β, respectively. Note that for some values of $(\Delta d_{12}, \gamma)$, subscriber utility (i.e. user surplus) is positive, but is always zero at market equilibrium. The following corollary generalizes this trend.

Corollary 1 (zero surplus)**.** *There is no path through the market equilibrium in which users have positive utility, i.e., user surplus is always zero. Thus, the MNOs will offer priority access so that the user's surplus becomes zero.*

9. Since a subgame equilibrium exists and is unique for each of the three stages, it follows that the market equilibrium is also unique.

Proof. This can be shown by evaluating the pricing equilibrium at the priority access equilibrium (Lemma 8 – Lemma 12 in Appendix B) and observing that the pricing equilibrium of Lemma 2 cannot be in any equilibrium path. ∎

Priority Access Incentives: Corollary 1 provides insight into how MNO 2 is incentivized to provide priority access. To highlight this insight, we walk through the market decisions which ultimately drive surplus to zero. Suppose that (contrary to Corollary 1) p_1, p_2, λ_1 and λ_2 are at the pricing equilibrium of Lemma 2 and thus the user surplus is positive. At this equilibrium, λ_1 and λ_2 do not depend on Δd_{12}. MNO 2 can exploit this fixed market share by using priority access to *increase* the QoS that MNO 1 provides to its subscribers. Although counterintuitive, this strategy benefits MNO 2 in the following two ways. First, by increasing MNO 1's QoS (i.e., $QoS_1 + QoS_{12}$) relative to its own QoS (i.e., QoS_{12}), the relative value of MNO 1's service increases, where it follows that MNO 1 can increase its service prices to reflect the increase in service value. Second, the increase of its competitor's service price also allows MNO 2 to increase its price, albeit a comparatively smaller increase in the latter case. Note that the QoS provided by MNO 2 is actually decreasing in Δd_{12}. Since λ_2 is constant here, this decrease does not affect MNO 2's profit. *Hence, when user surplus is positive, MNO 2 adopts a priority access strategy that differentiates its provided QoS from that provided by MNO 1 in order to increase its service price and drive surplus to 0.* This differentiation allows MNO 2 to better compete in the service market with MNO 1.

Impact on end-user: Corollary 1 also provides insight into how priority access impacts the user. We can characterize this impact in terms of service prices, user surplus, and QoS. So far, we have observed that priority access drives user surplus to zero and can result in increased service prices. While these effects harm the user, we find that priority access can have some positive impact on the user in terms of QoS. This positive impact is summarized by the following corollary.

Corollary 2 (maximized total QoS)**.** *When user surplus is zero, MNO 2 uses priority access to maximize the total QoS provided*

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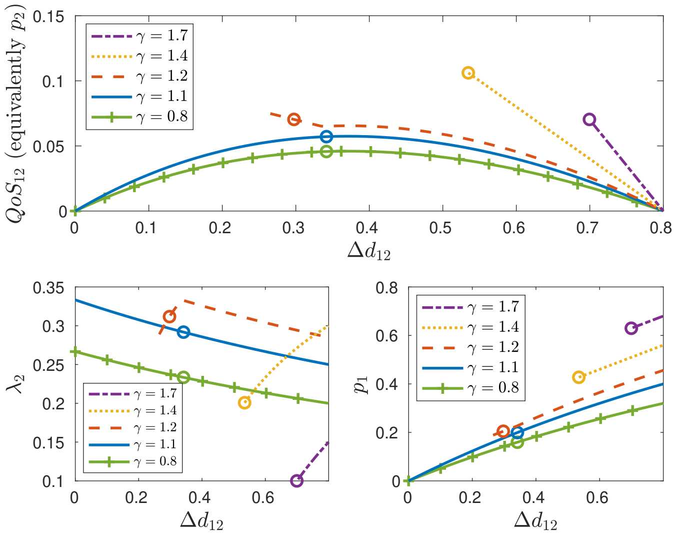

Fig. 6: Normalized QoS_{12} and λ_2 at the second stage equilibrium plotted where $u_2 = u_1 = 0$. $d_1 = 0$, $d_{12} = 0.8$. Circular line markers indicate the priority access equilibrium.

to its subscribers, i.e., $\lambda_2 \Lambda QoS_{12}$.

Proof. By (3), zero surplus implies that $vQoS_{12} = p_2$. Since Δd^*_{12} maximizes profit $\pi_2 = \lambda_2 p_2 - q_2^{total}$, Δd^*_{12} also maximizes $\lambda_2 \Lambda QoS_{12}$. ■

By maximizing total QoS, MNO 2 provides the highest possible QoS to the largest number of users. This behavior is the result of MNO 2 using priority access to differentiate its QoS with MNO 1 where differentiation is achieved by maximizing total QoS. If we take QoS instead of total QoS, we can draw the following conclusion.

Remark (nearly maximized QoS)**.** *When user surplus is zero, MNO 2 uses priority access to nearly maximize* QoS_{12} *for its subscribers. In some cases, this is achieved by shrinking its market share by more than 30 % compared to no priority access. Evidence for this remark is provided in Appendix D.*

Combining the last several observations, we can draw the following conclusions on how medium-term sharing impacts the user. On the positive side, sharing benefits the user by maximizing QoS delivered to subscribers of MNO 2. On the negative side, sharing can harm the user by increasing service prices and driving user surplus to zero. In some cases, users may be pushed out of the market. In Section 5.4, we take a more holistic view of the user impact question by considering our NS framework from a regulatory perspective.

5.4 Comparisons

We compare service market outcomes under our framework to the following three baselines: when MNOs 1 & 2 share the network but offer no further access (i.e., $\Delta d_{12} = \Delta d_1 = 0$), when the fixed cost constraint is relaxed, and when MNO 1 monopolizes the market. These baselines will help us evaluate the regulatory and economic viability of our network sharing framework. First, we characterize the monopoly equilibrium.

Notation	Description	Relation to Pricing Equil.
p_i^*	MNO i's service price under network sharing framework	$p_i^* = p_i\big\vert_{\Delta d_{12}=\Delta d^*_{12}}$
$p_{i,na}$	MNO i's service price under no access	$p_{i,na} = p_i\big\vert_{\Delta d_{12}=0}$
p_{mono}	service price under monopoly	$p_{mono} = p_1\big\vert_{\Delta d_{12}=d_{12}}$
$p_{i,relax}$	MNO i's service price under relaxed fixed cost constraint	$p_{1,relax} = p_{mono}$,
$p_{i,rr}$	MNO i's service price under running royalties	simulated,
Σ^*_{QoS}	average QoS under NS framework	$\Sigma^*_{QoS} = \Sigma_{QoS}\big\vert_{\Delta d_{12}=\Delta d^*_{12}}$
$\Sigma_{QoS,na}$	average QoS under no access	$\Sigma_{QoS,na} = \Sigma_{QoS}\big\vert_{\Delta d_{12}=0}$
$\Sigma_{QoS,mono}$	average QoS under monopoly	$\Sigma_{QoS,mono} = \Sigma_{QoS}\big\vert_{\Delta d_{12}=d_{12}}$
$\Sigma_{QoS,relax}$	average QoS under monopoly	$\Sigma_{QoS,relax} = \Sigma_{QoS,mono}$
$\Sigma_{QoS,max}$	maximum average QoS over all $\Delta d_{12} \in [0, d_{12}]$	$\Sigma_{QoS,max} = \max_{\Delta d_{12}} \Sigma_{QoS}$
$\Sigma_{QoS,rr}$	average QoS under running royalties	Simulated
Σ^*_{Λ}	total no. of subs. under NS framework	$\Sigma^*_{\Lambda} = \Sigma_{\Lambda}\big\vert_{\Delta d_{12}=\Delta d^*_{12}}$
$\Sigma_{\Lambda,na}$	total no. of subs. under no access	$\Sigma_{\Lambda,na} = \Sigma_{\Lambda}\big\vert_{\Delta d_{12}=0}$
$\Sigma_{\Lambda,mono}$	total no. of subs. under monopoly	$\Sigma_{\Lambda,mono} = \Sigma_{\Lambda}\big\vert_{\Delta d_{12}=d_{12}}$
$\Sigma_{\Lambda,relax}$	total no. of subs. under relaxed fixed cost constraint	$\Sigma_{\Lambda,relax} = \Sigma_{\Lambda,mono}$
$\Sigma_{\Lambda,max}$	maximum no. of subs. over all $\Delta d_{12} \in [0, d_{12}]$	$\Sigma_{\Lambda,max} = \max_{\Delta d_{12}} \Sigma_{\Lambda}$
$\Sigma_{\Lambda,rr}$	total no. of subs. under running royalties	simulated

TABLE 1: Description of notations and their relationship to the pricing equilibrium.

5.4.1 Monopoly Baseline

In the monopoly baseline, we suppose that MNO 1 is the only service provider. In the scenario, MNO 1 uses both its individual network deployment and the shared network deployment to support services for its subscribers.

Lemma 6 (monopoly equilibrium)**.** *Suppose that MNO 1 is the only service provider and let* p_{mono} *and* λ_{mono} *be the service price and market share at equilibrium. Then for* $\gamma \geq 2$, $p_{mono} = (d_1 + d_{12})(v - \beta)$ *and* $\lambda_{mono} = 1$. *Otherwise, if* $\gamma < 2$, *then* $p_{mono} = (d_1 + d_{12})v/2$ *and* $\lambda_{mono} = \gamma/2$.

The above Lemma follows from Lemma 1 through Lemma 4 when $s_{12} = 0$.

5.4.2 Relaxed Cost Constraint Baseline

The second baseline we consider is when MNO 1 and MNO 2 share the network and the fixed cost constraint is relaxed. For this baseline, we suppose that cost q_i^{total} for $i = 1, 2$ may vary as an arbitrary function of Δd_{12} and Δd_1. W.L.O.G., let $q_1^{total} = q_1 + \Delta q_{12} + t_{12} - t_{21}$ and $q_2^{total} = q_{12} - \Delta q_{12} + t_{21} - t_{12}$ where MNO 1 pays MNO 2 a price $t_{12} \geq 0$ for priority access Δd_{12}, MNO 2 pays MNO 1 a price $t_{21} \geq 0$ for primary access

Δd_1. Hence, q_1^{total} and q_2^{total} are not fixed costs. In this scenario, we modify the first stage game such that MNO 1 chooses t_{21} and Δd_1, and MNO 2 chooses t_{12} and Δd_{12}. Hence, t_{12} and t_{21} are *strategic choices*. We assume that MNO 1 and MNO 2 cooperatively bargain for access and prices. The following lemma characterizes the resulting market equilibrium.

Lemma 7 (relaxed fixed cost constraint)**.** *Suppose that costs q_1^{total}, q_2^{total} are not fixed and let $p_{1,relax}$ and $\lambda_{1,relax}$ be the service price and market share of MNO 1 at equilibrium. Then there is an equilibrium where $p_{1,relax} = p_{mono}$ and $\lambda_{1,relax} = \lambda_{mono}$.*

Proof. At one possible equilibrium (e.g., the Nash bargaining solution [51]), MNO 2 chooses Δd_{12} to maximize the sum profit $\pi_1 + \pi_2$ (after which the MNOs strategically choose t_{12} and t_{21} to split the maximized sum profit). Our goal is to show that $\Delta d_{12} = d_{12}$ maximizes $\pi_1 + \pi_2$. We first observe that $\pi_1 + \pi_2$ is a convex function of Δd_{12}. To see this, the reader can verify that $\pi_1 + \pi_2$ (when evaluated at the pricing equilibrium Lemma 1 through Lemma 4) always has a non-positive second derivative (w.r.t. Δd_{12}). Furthermore, we can show that

$$\left.\frac{\partial(\pi_1 + \pi_2)}{\partial \Delta d_{12}}\right|_{\Delta d_{12} = d_{12}} \geq 0. \quad (6)$$

Together, the convexity of $\pi_1 + \pi_2$ and (6) imply that $\pi_1 + \pi_2$ is non-decreasing in $\Delta d_{12} \in [0, d_{12}]$. Hence, $\pi_1 + \pi_2$ is maximized at $\Delta d_{12} = d_{12}$ which coincides with a monopoly. ■

5.4.3 No Access Baseline

The next baseline we consider is when MNO 1 and MNO 2 share the network with no additional access to resources, i.e., $\Delta d_{12} = \Delta d_1 = 0$. The equilibrium for this baseline is given by the pricing equilibrium Lemma 1 through Lemma 4 evaluated at $s_1 = d_1$ and $s_{12} = d_{12}$. This baseline captures the network sharing scenario considered in a few prior works [41], [42].

5.4.4 Running Royalties Baseline

The last baseline we consider is when MNO 1 and MNO 2 share the network and and shared costs are allocated based on each MNO's output. This baseline was studied in [20] under the name *running royalties* and is a special case of the relaxed fixed cost constraint baseline. For the running royalties baseline, we set the cost transfer so that each MNO pays a fraction of the shared cost q_{12} proportional to their number of subscribers, i.e., $\Delta q_{12} = q_{12}\frac{\lambda_1}{\lambda_1+\lambda_2}$.[10] Thus, $q_1^{total} = q_1 + q_{12}\frac{\lambda_1}{\lambda_1+\lambda_2}$ and $q_2^{total} = q_{12}\frac{\lambda_2}{\lambda_1+\lambda_2}$. We remark that equilibrium depends on q_{12}, and thus a value of q_{12} must be chosen to simulate the equilibrium. We choose a value of $q_{12}/\Lambda = 0.15$ to reflect the average net profit margin (7 %) of the telcom services sector [52].[11]

5.4.5 Comparison Metrics

We use the following metrics to assess the regulatory and economic viability of our NS framework. To assess the regulatory viability of our NS framework, we check whether the NSA meets certain regulatory objectives defined by the Body of European Regulators for Electronic Communications (BEREC). According to the BEREC, an NSA should maintain the following two regulatory objectives: effective *competition* and efficient network *utilization* [53].[12] It is important to consider both competition and network utilization when assessing whether an NSA benefits or harms the end-user.

As a metric of competition, we use service prices p_1 and p_2. Prices are arguably the most important metric in assessing competition. We say that an NSA maintains effective competition when service prices are low compared to the baselines. As a metric of network utilization, we use the total number of subscribers $\Sigma_\Lambda = (\lambda_1 + \lambda_2)\Lambda$ and the average QoS per user $\Sigma_{QoS} = \lambda_1(QoS_1 + QoS_{12}) + \lambda_2 QoS_{12}$. We say that an NSA efficiently utilizes resources when both Σ_Λ and Σ_{QoS} are large compared to the baselines. We denote the above metrics under our framework with a $*$ superscript. We denote the above metrics under the no access, relaxed fixed cost constraint, monopoly, and running royalties baseline with a 'na', '$relax$', '$mono$', and 'rr' subscript, respectively. The metrics for each baseline are summarized in Table 1.

To assess the economic viability of our NS framework, we compare the profitability of our NS framework to the no access baseline. This comparison allows us to evaluate how profitable it is for two MNOs to adopt our framework given that their current NSA does not allow dynamic sharing in the medium-term. We say that our framework is profitable if the service revenue of both MNOs is large compared to the no access baseline.

5.4.6 Results

The following result compares regulatory objectives of an NSA under our NS framework to the baselines.

Theorem 2 (comparison of regulatory objectives)**.** *For $i = 1, 2$, service price p_i^* of MNO i under our network sharing framework is upper bounded such that*

$$p_i^* \leq p_{1,relax} = p_{mono}. \quad (7)$$

The average QoS Σ_{QoS}^* and total number of subscribers Σ_Λ^* under our network sharing framework are bounded such that

$$\Sigma_{QoS,na} \leq \Sigma_{QoS}^* \leq \Sigma_{QoS,max} = \Sigma_{QoS,relax} = \Sigma_{QoS,mono} \quad (8)$$

and

$$\Sigma_{\Lambda,relax} = \Sigma_{\Lambda,mono} \leq \Sigma_\Lambda^* \leq \Sigma_{\Lambda,max} = \Sigma_{\Lambda,na}. \quad (9)$$

Proof. Equation (7) follows from 3 observations: 1) $p_2 \leq p_1$, 2) p_1 is non-decreasing in Δd_{12}, and 3) p_1 evaluated at $\Delta d_{12} = d_{12}$ is equal to p_{mono}. Equation (8) (equation (9)) follows from the observation that Σ_{QoS} (Σ_Λ) is non-decreasing (non-increasing) in Δd_{12}. ■

Effective Competition: Theorem 2 shows that, in general, service prices under our framework are smaller than under the relaxed fixed cost constraint baseline and monopoly baseline. Thus, we can conclude that our NSA framework maintains effective competition compared to a monopoly setting. This conclusion, however, may not be sufficient to convince a regulator that the framework maintains effective competition. A regulator may be concerned that framework distorts competition compared to an NSA *without medium-term sharing*,

10. No access is granted, i.e., $\Delta d_{12} = 0$.

11. The value $q_{12}/\Lambda = 0.15$ is the solution to $0.07 = \frac{revenue-cost}{revenue} = \frac{\lambda_1 p_1 \Lambda - q_{12}}{\lambda_1 p_1 \Lambda}$ where $\lambda_1 p_1 \approx 0.16$ at the no access equilibrium.

12. The BEREC also considers connectivity as an objective. However, in our assessment, we omit this objective as connectivity is not well-defined in our network model.

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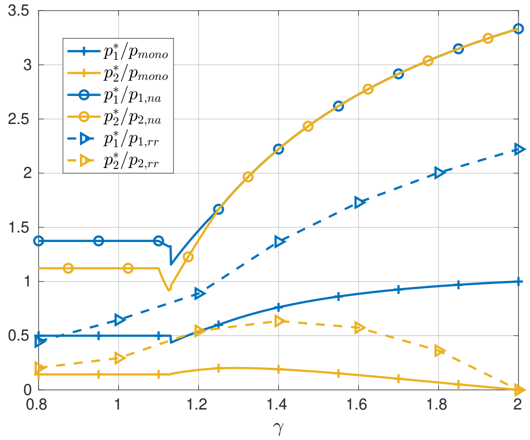

Fig. 7: service prices p_i^* under network sharing framework normalized by monopoly prices and no access prices. $d_1 = 0.3$ and $d_{12} = 0.7$.

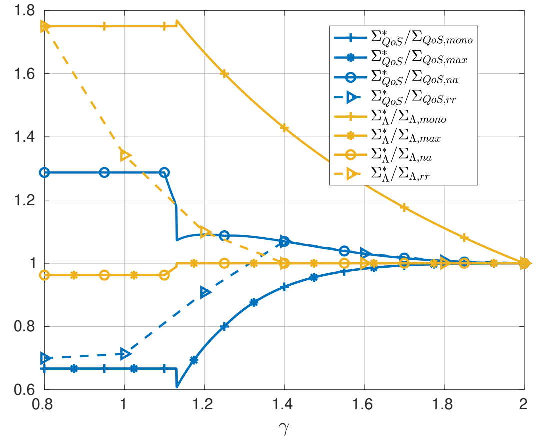

Fig. 8: Average QoS Σ_{QoS}^* and total number of subscribers Σ_{Λ}^* under network sharing framework normalized by their respective monopoly quantity, no access quantity, and max quantity. $d_1 = 0.3$ and $d_{12} = 0.7$.

i.e., the no access baseline. To address this setting, we turn to numerical evaluations which are illustrated in Fig. 7.

Fig. 7 shows that service prices under our framework can be much larger than under no access baseline. Indeed, service prices can be more than 3 *times larger* depending on the value of the isolation factor γ.[13] Thus, a regulator may conclude that the framework stifles competition. However, to ensure that their NSA maintains effective competition, MNOs can choose a value of γ that yields low services prices. For example, Fig. 7 shows that this value may be closer to 1 than to 2. *We expect that some negotiation between MNOs and regulators will be needed to choose an appropriate value of γ which will allow the NSA to maintain effective competition.*

Efficient Utilization: Similar to the above competition metrics, our results show that the network utilization metrics depend on γ. Fig. 8 shows that the total number of subscribers under our framework can be much larger compared to the monopoly setting and even reach the maximum value for large enough γ. Furthermore, average QoS under all baselines becomes tight as γ approaches 2. Thus, a regulator may conclude that the framework maintains effective utilization for large enough γ. Comparing this result to the results so far, we find the following tradeoff in γ between effective competition and efficient utilization: *smaller γ yields more competitive prices and larger γ yields more efficient network utilization.* Guidance from regulators may be needed to find the right balance in this tradeoff.

Economic Viability To assess the profitability of our framework, we compare MNO revenue before and after they adopt our NS framework. While this comparison can be performed for all γ, we provide a more concise comparison by choosing a specific value of γ and comparing revenues for that given value. We consider the case where MNOs choose γ to jointly optimize their individual profits. That is, we choose γ to be the Pareto optimal solution γ^* to the problem $\max_{\gamma>0}(\pi_1, \pi_2)$. Note that the value of γ^* may differ depending on whether the MNOs use our framework or provide no access.

13. A similar trend appears for the running royalties baseline, although this trend may be less of a concern for regulators since p_1^* is much less than $p_{1,rr}$ and thus MNO 1's prices are more competitive under our framework than under the running royalties baseline.

Theorem 3 (comparison of revenue)**.** *Suppose that $d_{12} \geq d_1$ and $\gamma = \gamma^*$. Then the revenue of MNO i under our framework is larger than its revenue under the no access baseline by a factor of*

$$\frac{9(d_1 + d_{12})(d_1 + 3d_{12})^2}{32d_1(d_1 + 2d_{12})^2} \tag{10}$$

for $i = 1$ and a factor of

$$\frac{(d_1 + d_{12})(d_1 + 3d_{12})^2}{32d_1 d_{12}^2}, \tag{11}$$

for $i = 2$.

Proof. We first characterize γ^*. When no access occurs, we observe that π_1 is increasing in γ. It follows that $\gamma^* = \arg\max_{\gamma>0} \pi_2$. Hence, γ^* can be shown to be $(2d_1 + 3d_{12})/(d_1 + 3d_{12})$ by solving the corresponding optimization problem at the pricing equilibrium (Lemma 2 through Lemma 4) at $\Delta d_{12} = 0$. Under our framework, π_1 is not increasing in γ. However, we observe that $\arg\max_{0<\gamma\leq 5/4} \pi_1$ is equal to $5/4$ by solving the problem where π_1 is evaluated at the pricing equilibrium at $\Delta d_{12} = \Delta d_{12}^*$ where Δd_{12}^* is given by Lemma 9 through Lemma 12. Furthermore, $\arg\max_{\gamma>0} \pi_2$ can be shown to be $5/4$. Hence, under our framework, $\gamma^* = 5/4$.

To prove Theorem 3, we can show equation (10) and equation (11) by dividing MNO i's revenue $\lambda_i \Lambda p_i$ evaluated at $\Delta d_{12} = \Delta d_{12}$ and $\gamma^* = 5/4$ by its revenue evaluated at $\Delta d_{12} = 0$ and $\gamma^* = (2d_1 + 3d_{12})/(d_1 + 3d_{12})$. ■

Theorem 3 shows that it can be profitable for MNOs to adopt our framework given that their current NSA does not allow medium-term sharing. Furthermore, Theorem 3 provides insight into how profitable our framework is over various stages of the 5G rollout. We broadly consider two stages: early 5G and mature 5G. Early 5G services (e.g., enhanced mobile broadband (eMBB)) are largely similar to 4G services. In fact, 4G networks may be able to offer the 5G-like performance that early 5G services require, especially as some MNOs continue to improve their LTE networks (e.g., LTE Plus, 5G Evolution). As 5G matures and new services emerge that have performance requirements exceeding 4G capabilities, legacy networks will prove to be weaker substitutes for 5G networks.

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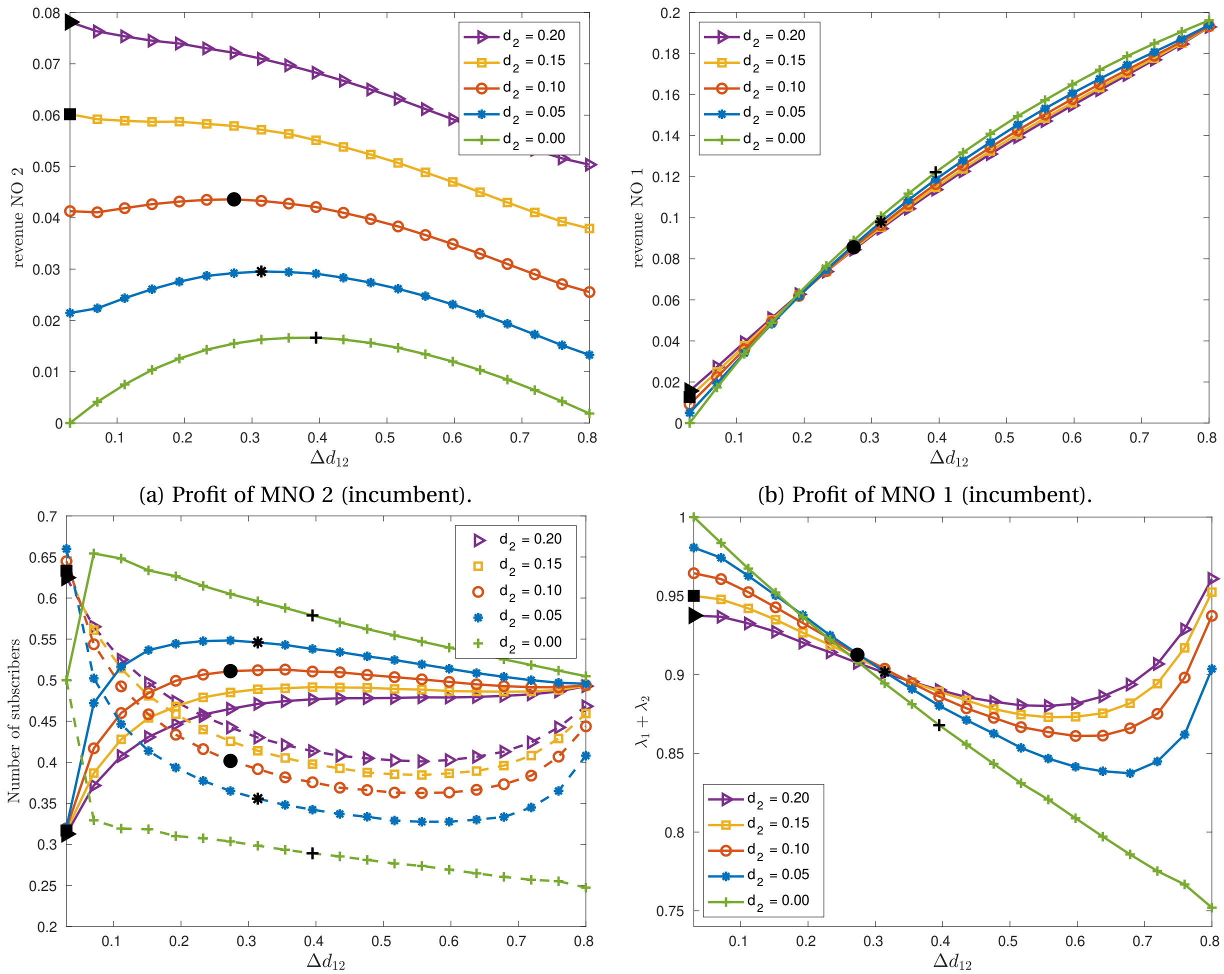


(a) Profit of MNO 2 (incumbent).

(b) Profit of MNO 1 (incumbent).

(c) Market shares λ_1 (solid lines) and λ_2 (dashed lines).

(d) Total share of market subscribing to an MNO.

Fig. 9: Two incumbent MNOs. Isolation factor $\gamma = 1$. $d_1 = 0$ and $d_{12} = 0.8$. Solid black line markers indicate the priority access strategy Δd_{12}^* that maximizes MNO 2 profit.

In terms of our model, this trend implies that the guaranteed demand d_1 of MNO 1's legacy network will decrease over time as 5G rollouts transition from early 5G to mature 5G. Assuming that the joint venture between MNO 1 and MNO 2 is responsible for the 5G rollout as well as the transition from early 5G to mature 5G, the guaranteed demand d_{12} of the shared 5G network can be expected to stay constant or increase during this time. Correspondingly, Theorem 3 shows that our framework will be more profitable over time, as both factor (10) and factor (11) are increasing as d_1 decreases.

6 Two Incumbent MNOs

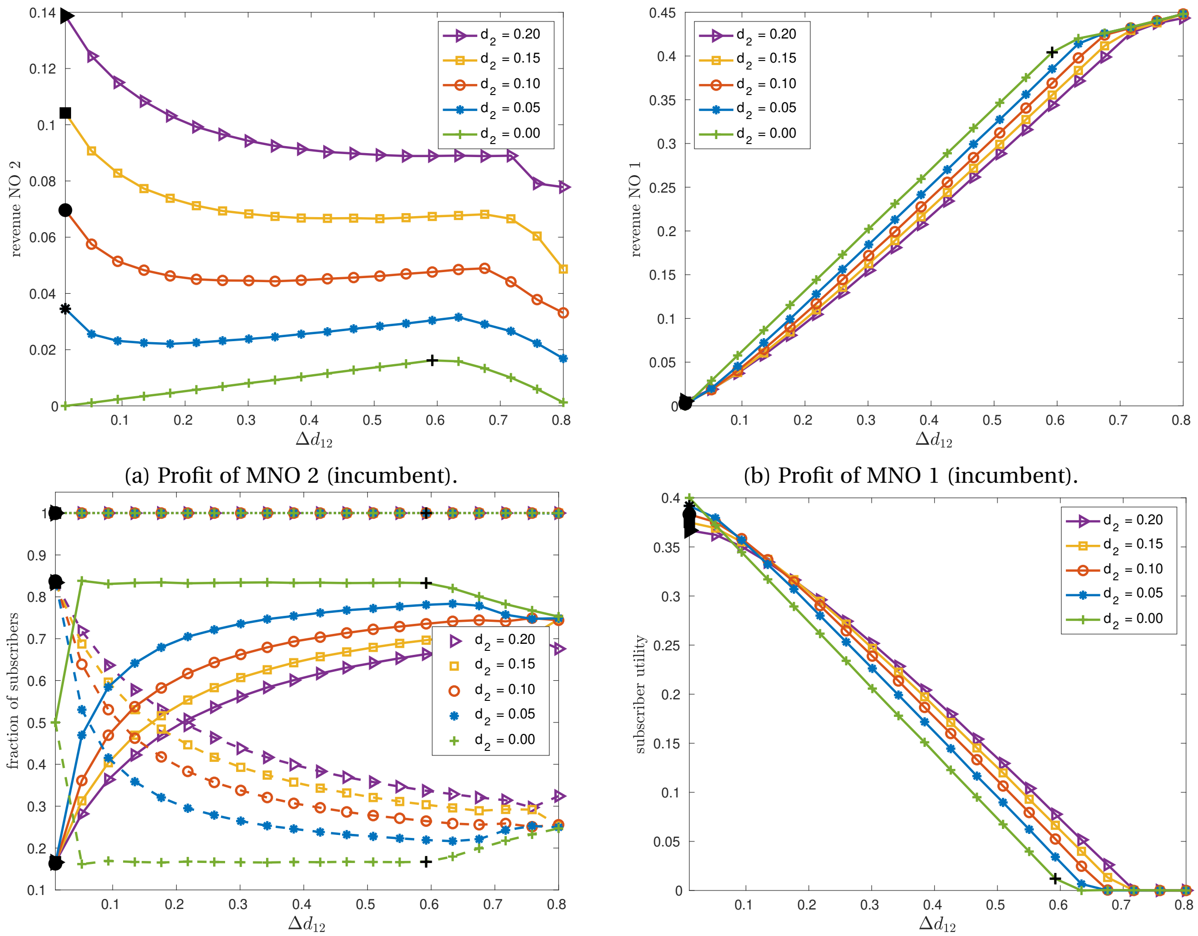
This section investigates an extension of our framework from network sharing between an incumbent MNO and new entrant MNO to network sharing between 2 incumbent MNOs. We illustrate via simulations that our framework is not effective when both MNOs are incumbents. In the sequel, we redefine MNO 2 to be an incumbent MNO such that it has legacy 2G/3G/4G resources in the geographic area of focus. Like MNO 1, MNO 2 can use its legacy resources to serve its subscribers. We now extend the slicing model of Section 3.3 to account for these legacy resources.

We let MNO 2 use its legacy resources to orchestrate a subslice $SLICE_2$ with guaranteed demand $d_2 > 0$. Only users subscribed to MNO 2 will have their traffic serviced by this subslice. After accounting for the guaranteed demand lost to overbooking, i.e., $d_2 g(\lambda_2)$, the QoS of $SLICE_2$ can be verified to be $QoS_2 = d_2 - \alpha d_2 g(\lambda_2)$. We can now extend the subscribers' utility (4) to account for this extra QoS term. It follows that the utility of a subscriber to MNO 2 is $u_2 = v(QoS_2 + QoS_{12}) - p_2$. Note that the utility of a subscriber to MNO 1 is not affected by $SLICE_2$, and therefore, is still equal to utility (3).

In the simulations that follow, we simulate the market equilibrium for the service market consisting of MNO 1, MNO 2, and the user pool. Specifically, we simulate the second stage (and third stage) market equilibrium (defined in Section 4.4).[14] In the following figures, profit and utility are normalized by $\beta\Lambda$ and β, respectively.

We begin by simulating a market with a small isolation factor. Fig. 9 shows the resulting second stage market equilibrium when $\gamma = 1$. We consider $d_1 = 0$ and $d_{12} = 0.8$. Fig. 9a shows

14. The only non-trivial difference now is that we must exclude from the equilibrium path cases where QoS_{12} given by (2) is negative. Note that the condition $u_2 \geq 0$ was sufficient to prevent this from happening when MNO 2 was a new entrant.



(a) Profit of MNO 2 (incumbent).

(b) Profit of MNO 1 (incumbent).

(c) Market shares λ_1 (solid lines), λ_2 (dashed lines) and $\lambda_1 + \lambda_2$ (dotted lines).

(d) Utility of subscriber to MNO i for $i = 1, 2$.

Fig. 10: Two incumbent MNOs. Isolation factor $\gamma = 1.5$. $d_1 = 0$ and $d_{12} = 0.8$. Solid black line markers indicate the priority access strategy Δd_{12}^* that maximizes MNO 2 profit.

MNO 2's profit for various values for d_2. Unsurprisingly, we notice that profit is increasing in d_2. Here, an increase in d_2 results in an increase in MNO 2's total guaranteed demand, allowing MNO 2 to provide better QoS and increase its market share (where market share is depicted in Fig. 9c). However, some market share is unobtainable, as depicted in Fig. 9d. Since the isolation factor is small, a large number of QoS violations may occur. Therefore, some users will forgo the poor QoS and a fraction of the user pool remains unsubscribed.

As d_2 increases, we observe that MNO 2's priority access strategy changes from one that maximizes $QoS_2 + QoS_{12}$ to one that maximizes market share λ_2. *In other words, the conclusion of Remark 5.3 no longer holds.* To see how this happens, consider MNO 2's market share and the total share of the market subscribing to an MNO in Figs. 9c and 9d. Fig. 9d shows that as d_2 increases, the rate at which $\lambda_1 + \lambda_2$ decreases in Δd_{12} around $\Delta d_{12} = 0$ slows. Therefore, as MNO 2 offers a larger QoS_2 to the user pool, users become more inclined to stay in the market. This trend suggests that when MNO 2 is redefined from a new entrant to an incumbent, priority access is less effective at decreasing $\lambda_1 + \lambda_2$ and pushing users out of the market. Hence, for larger d_2, priority access becomes less effective at increasing QoS_{12}, which is a function of $\lambda_1 + \lambda_2$. Meanwhile, for $d_2 > 0$, priority access forces a sudden decrease in λ_2 around $\Delta d_{12} = 0$ as illustrated in Fig. 9c. Therefore, MNO 2 adopts a priority access strategy that maximizes its market share. This change in strategy has a detrimental impact on MNO 1's profit. Note that when d_2 is large enough, the priority access equilibrium Δd_{12}^* is 0. Fig. 9b shows that although variation in d_2 has a minimal effect on MNO 1's profit for fixed Δd_{12}, profit will plummet after we account for the fact that Δd_{12}^* decreases in d_2 from a positive quantity to zero.

Few of these observed market dynamics change when the isolation factor is increased to $\gamma = 1.5$, as depicted in Fig. 10. As d_2 increases (and MNO 2 optimizes profit over Δd_{12}), we again see MNO 2's profit increase and priority access decrease (see Fig. 10a) followed by a significant drop in MNO 1's profit (see Fig. 10b). Compared to the case when $\gamma = 1$, one notable difference here is the change in user behavior. Specifically, Fig. 10c illustrates that when d_2 and Δd_{12}, increases and decreases, respectively, a dramatic shift in market share occurs where the majority of users leave MNO 1 to subscribe to MNO 2. Furthermore, Fig. 10d shows that all users, regardless of their subscribed MNO, may see a large increase in utility. In contrast the conclusion of Corollary 1, it is now possible under reserved sharing for user surplus to be strictly positive.

These simulations suggest that when both MNOs are incumbent MNOs, the proposed network sharing framework does not always incentivize priority access. This conclusion is illustrated in Fig. 9 and Fig. 10. Hence, the framework is not always an effective solution for dynamically sharing resources in the medium-term when both MNOs are incumbents. However, as shown in the above figures, the framework does incentives priority access for some network parameters.

7 CONCLUSION

This article proposed a network sharing framework that allows dynamic resource sharing between an incumbent MNO and new entrant MNO under a fixed cost constraint. The framework allows sharing in the medium-term – a time frame where market shares are variable but network costs remain fixed. Via market equilibrium analysis, we characterized the effect of our framework on MNO revenue and various metrics of user welfare. We assessed the regulatory and economic viability of our framework. We conclude that our framework will increase in profitability as the 5G rollout matures and 5G services become more distinguished from legacy network services. Simulations suggest our framework may not be effective for sharing between two incumbent MNOs.

There are several directions in which this article can be extended. To address more practical markets, the framework can be extended for network sharing settings between 3 or more MNOs. The Authors conjecture that a careful extension of the framework can allow for effective sharing between a new entrant MNO and 2 incumbent MNOs. Evidence for this conjecture follows from an observation that the user utilities can have a similar form to the user utilities of a new-entrant/incumbent setting. Beyond a multi-MNO setting, user utilities can be remodeled to capture the heterogeneous nature of service valuations. The tools and insights we have developed will be useful to address these extensions.

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