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Robust Scalable Physical Layer Network Coding

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***Abstract*—We present a new method of physical layer network coding that transforms the received signal at a relay node before re-transmission by eliminating the unreliable components of the signal. This approach improves the physical layer network coding performance, since only reliable components of the signal are amplified and retransmitted. It also lends itself to a robust and scalable implementation. Simulation results show that the proposed robust scalable physical layer network coding can provide $1-2$[dB] and $4-14$ [dB] more coding gain than analog network coding and algebraic network coding schemes in a multi-layer wireless network, respectively, and performs more reliably in a network where there is more noise to remove.**

I. Introduction

In the most recent report from Cisco [22] and World Wide Research Forum [24], it is predicted that mobile traffic, the number of wireless devices, and their processing power are increasing along with usability improvements. This suggests that the capability and accessibility of resources for sharing across mobile devices also grows. For example, advances in ultra low power sensors devices will enable sharing of resources wirelessly to be more practical, realizable, and sustainable. Increases in user desire to save time and energy; size of metropolitan city area globally; network operators sharing resources; communities sharing content; live events for sharing experiences; and sensors for medical, environment, and safety create a fertile ground for enhanced applications that will increase efficiencies and reward cooperative behaviors. Such emerging future networks applications include wearable wireless body area networks, close cooperative communities providing direct device-to-device communications without infrastructure in a nearby neighborhood or community, inter/intra-vehicle sensor networks, and smart homes.

For such consumer applications, a wireless ad-hoc network needs to accommodate multiple data-flows that have different source/destination pairs. Each data-flow takes multiple hops over the air from a source node to a destination node, thus requiring intermediate nodes for relaying. Usually, such relaying protocol has been implemented using ad-hoc routing protocols, for example, ad-hoc on demand vector routing (AODV) in WLAN mesh using IEEE 802.11s standards [23]. Unfortunately, such ad-hoc routing schemes may not provide enough throughput and robustness to support consumer-grade handheld devices. This is because multiple data-flows would intersect at a certain relay node, the ad-hoc routing protocols may suffer from the bottleneck effect at the relay node. In addition, since handheld devices would frequently move in and out of the network, and packet loss and link failures would occur at unknown locations, ad-hoc routing protocols may not support sufficient robustness to such topology changes and may increase error rates. One of the candidates for improving ad-hoc routing protocols is network coding [7].

It has been shown (see, for instance, [2], [10], [11], [15]) that network coding can provide significant advantages, particularly in multicast setting. By network coding, the relay nodes in a multihop setting could do more than simply forwarding the underlying packets. In algebraic network coding, various packets arriving at a node can be combined (for instance by the application of simple XOR operation or by forming a linear combination of the packets over a finite field) into a *network coded packet*, and this combination can then be forwarded to destinations. If there are enough network coded packets at each destination node to solve for the original packets of interest, then the original packets can be re-constructed.

In the above, if the network coded packets are formed from complex field functions of the physical layer versions of the underlying packets, we say that physical layer network coding is being performed. Physical layer network coding is particularly of interest because the wireless networks have broadcast nature of medium and multi-hop with multiple relay nodes as their characteristics. If multiple sources transmit to a receiver node in a synchronized manner, then the received signal can be formed into a physical layer coded version of the transmitted packets by some process. This gives some importance to the design of physical layer network coding strategies.

A. Related Work

This problem has been mainly studied for a bidirectional channel with two communicating nodes that employ a relay node for communicating to each other over the same channel (see [13] and reference therein). For uni-directional communications, there are some prior work [6], [17] that employ lattices. Consider a scenario that a node v_j receives source packets $P_{1,j}, P_{2,j}, \cdots, P_{k,j}$ from nodes v_i, $i = 1, 2, \cdots, k$ with average transmit power P per transmitting node. Let the packet $P_{i,j}$ be encoded by l constellation symbols

$$X^1_{i,j} X^2_{i,j} \cdots X^l_{i,j},$$

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and assume the received word at v_j be given by

$$Y_j^1 Y_j^2 \cdots Y_j^l.$$

The received packet Y_j^m, $m = 1, 2, \cdots, l$ is modeled as the corrupted version of scaled versions of $X_{i,j}^m$ by n_j^m i.i.d. samples of the additive noise n_j with mean zero and variance σ^2 per complex dimension.

In mathematical notation

$$Y_j^m = \sum_{i=1}^{k} \alpha_{i,j} X_{i,j}^m + n_j^m, \tag{1}$$

for $m = 1, 2, \cdots, l$, where $\alpha_{i,j}$ denote the channel from node v_i to the node v_j.

In both [6] and [17], the signals $X_{i,j}^1 X_{i,j}^2 \cdots X_{i,j}^l$ for $i = 1, 2, \cdots, k$ are chosen from the same lattice L. In [17], the selected lattice is the integer lattice $\mathbb{Z}^l$. In [6], other lattices with higher shaping gains are allowed. According to [6], [17], node v_j computes a summation $\tilde{Y}_j^m = \sum_{i=1}^{k} b_i X_{i,j}^m$ for $m = 1, 2, \cdots, l$, where $(b_1, \cdots, b_k)$ is an integer vector. It then transmits

$$\tilde{Y}_j^1 \tilde{Y}_j^2 \cdots \tilde{Y}_j^l.$$

Clearly the transmitted vector also belongs to the lattice L.

To construct the transmitted vector, the node v_j first chooses a factor $\lambda \neq 0$ such that $(\lambda\alpha_{1,j}, \lambda\alpha_{2,j}, \cdots, \lambda\alpha_{k,j})$ is close to a point $(b_1, \cdots, b_k)$ of the integer lattice $\mathbb{Z}^k$ (In [6], the restriction of using the $\mathbb{Z}^k$ lattice is further relaxed and other lattices are used). If $(b_1, \cdots, b_k)$ is the closest point of the integer lattice to $(\lambda\alpha_{1,j}, \cdots, \lambda\alpha_{k,j})$, then the signal to noise ratio of the transmitted signal at node v_j is given by

$$\frac{P}{\lambda^2 + \sum_{i=1}^{k} P|\lambda\alpha_{i,j} - b_i|^2}.$$

Thus λ must be chosen so that the above signal to noise ratio is maximized. The main issue with the above approach is that if the coefficients $\alpha_{i,j}$ are not perfectly known at node v_j, then if a large choice of λ in the above is required, the actual signal to noise ratio of the transmitted signal can be small. Thus, we will need to produce other physical layer network coding schemes, which are robust, scalable and local, i.e., do not require too much knowledge of network topology at each node. This motivates our work in this paper.

B. Contributions

Our main contribution is a new physical layer network coding method based on removal of unreliable components of signals at each node. Based on a detailed analysis, we develop methods to achieve this task. Our physical layer network coding approach lends itself to a robust, scalable implementation. It is also local in the sense that each node only needs to know its neighbors.

The rest of the paper is organized as follows. In Section II, we will discuss our mathematical network model and establish the notation. In Section III, we will present the main idea of our approach. In Section IV, we will disclose our detailed approach to physical layer network encoding/decoding. Additionally, we will disclose how the proposed scheme can provide robustness and scalability. In Section V, we will provide simulation results demonstrating the performance of our scheme. Finally, Section VI concludes the paper and presents some future directions.

II. The Network Model

We consider a network with sources $S_1, S_2, \cdots, S_N$, relay nodes, and destination nodes $T_1, T_2, \cdots, T_M$. Consider a network graph G where vertices correspond to the network nodes. Two nodes v_i and v_j are connected by a directed edge to each other at time t if v_i can be heard at node v_j . Let $c_{ij}(t)$ be equal to the capacity of the link between v_i and v_j at time t. Note that this model capture time-varying physical channel models.

The main scenario of interest here is a multicast scenario where the packets $P_1, P_2, \cdots, P_N$, respectively, generated at nodes $S_1, S_2, \cdots, S_N$ must be received at all underlying destination nodes. The channels between nodes can be either modeled as discrete, Gaussian, or fading channels. Although our proposed construction method will apply to all these channels, in wireless scenarios Rayleigh or Rician fading channels may be of higher interest.

We consider a multi-layer wireless network that is divided into layers $0, 1, 2, \cdots$ corresponding to various stage of transmission of source packets to the destination nodes. Layer 0 consists of the source nodes, while layer $i \geq 1$ consists of the nodes that hear transmissions of the nodes in layer $i - 1$. The source packets are routed through the network layers in ascending order (layers $0, 1, \cdots$) until they arrive at the destination nodes. It could be assumed that all the nodes of layer i transmit simultaneously, i.e., over the same time-frequency-space resource, to elements of layer $i + 1$. It may also be assumed that some of the nodes at layer i transmit in different instances of time to elements of layer $i + 1$. This in particular can be useful if we want to produce multiple independent combinations of transmitted packets at each receiver node.

We assume that the underlying channels do not change very rapidly during the transmission of the packet and each receiver node v_j knows both the codebooks and the pilot signals of the packets transmitted by source nodes. The received packets at each node will be designed to be a linear sum of the packets transmitted by source nodes perturbed by noise. We refer to the linear coefficients of this linear sum as the effective channels from source nodes to receiver node v_j. Using the knowledge of the pilots of each source packet, the node v_j can estimate the underlying effective channels for its received packet.

As mentioned above, we focus on physical layer network coding. Nevertheless, we are inspired by the existing literature that are mostly focused on algebraic network coding. In particular, we are very interested in robustness [3]–[5], [8], [14], [16], [18]–[21], and scalability [12] of our proposed method.

III. Motivating Examples

We first discuss the main idea behind our approach.

A. Key Observation Motivating Our Approach

Our method of physical layer network coding is motivated by a simple observation given in the lemma below.

Lemma 1. *Consider the superposition*

$$Y = \alpha_1 X_1 + \alpha_2 X_2 + n$$

of signals X_1 and X_2 corrupted by noise as given by the network coding model described above. Suppose without loss of generality that $|\alpha_1| > |\alpha_2|$, and that X_1 and X_2 have average transmit power P. Suppose that X_1 can be determined from Y with high reliability by joint detection, but X_2 can be determined with low reliability. Consider the noisy signal $\tilde{Y} = \alpha_1 X_1 + n$. Consider an additive white Gaussian noise (AWGN) channel C with average input transmit power P. Then

- *If a scaled version of Y (with average power P) is sent through the channel C, the reliability of detecting X_2 from the output of the channel C is less than or equal to that of detecting X_2 from Y.*
- *Suppose that scaled versions of $\tilde{Y}$ and Y are sent through the channel C both scaled to have average power P, and the corresponding channel outputs are respectively $\tilde{Z}$ and Z. Then the reliability of detecting X_1 from $\tilde{Z}$ is in average greater than or equal to that from Z.*

Proof: The first statement follows trivially from the fact that transmission through the additive Gaussian channel only decreases the signal to noise ratio for X_1, X_2 and $\alpha_1 X_1 + \alpha_2 X_2$. In terms of information theory, it is also a consequence of the data processing inequality. The second part follows from the fact that by removing $\alpha_2 X_2$, the power of scaled version of X_1 in $\tilde{Y}$ is larger than that in Y. This means that the signal to noise ratio in $\tilde{Z}$ is larger than Z. ■

This motivates us to process the received signals in a manner that maintains the decoding reliability at the destination nodes. The goal of this processing is to increase the power of desired signal at each stage, thereby increasing the underlying signal to noise ratios at the destination nodes.

We next describe our basic approach by a simple example.

B. Illustration of Our Basic Approach by a Simple Example

The essence of our method is symbol by symbol processing and possible transformation of the received signals at each node. In order to put our construction in focus, we first compute the log-likelihood ratio for a scenario where two nodes v_1 and v_2 are respectively sending bits b_1 and b_2 simultaneously to a node v_3 using the BPSK constellation over a complex AWGN channel. Let the channel coefficient between the nodes i and j be $\alpha_{i,j}$. The received signal at node v_3 is modeled by

$$r = \alpha_{1,3} x_1 + \alpha_{2,3} x_2 + n,$$

where $x_i = (-1)^{b_i}$ for $i = 1, 2$ and the noise n has mean zero and variance $\sigma^2/2$ per real dimension.

Assuming that $b_i = 0$ and $b_i = 1$ have equal a priori probabilities for $i = 1, 2$, the log-likelihood of the bits b_1 and b_2 given the received signal is easily computed as

$$\begin{aligned} \text{LLR}(b_1) &= \ln \frac{E(\alpha_{1,3} + \alpha_{2,3}) + E(\alpha_{1,3} - \alpha_{2,3})}{E(-\alpha_{1,3} + \alpha_{2,3}) + E(-\alpha_{1,3} - \alpha_{2,3})} \\ \text{LLR}(b_2) &= \ln \frac{E(\alpha_{1,3} + \alpha_{2,3}) + E(-\alpha_{1,3} + \alpha_{2,3})}{E(\alpha_{1,3} - \alpha_{2,3}) + E(-\alpha_{1,3} - \alpha_{2,3})}, \end{aligned}$$

where

$$E(x) = \exp\left(-\frac{|r - x|^2}{\sigma^2}\right). \quad (2)$$

1) The Core of Our Approach: In the above, suppose that $\text{LLR}(b_1)$ is above an acceptable threshold but $\text{LLR}(b_2)$ is not. Then we will construct a new signal $\tilde{r}$ corresponding to the transmission model

$$\tilde{r} = \alpha_{1,3} x_1 + \tilde{n},$$

where $\tilde{n}$ is a zero mean AWGN noise with variance σ_2^2 per complex dimension. First, σ_2^2 is an unknown parameter to be optimized so that, for the scenario where b_1 is transmitted over a channel with gain $\alpha_{1,3}$ and additive noise $\tilde{n}$, and $\tilde{r}$ is received, $\tilde{r}$ produces in the Kullback-Leibler sense the approximately same log-likelihood distribution as $\text{LLR}(b_1)$. Second, the construction of $\tilde{r}$ will be performed so that it contains as little noise as possible, i.e. the expected power of the noise is as little as possible.

In this manner, $\tilde{r}$ eliminates any effect of the unreliable bit b_2 over the Gaussian noise channel in the above while reducing the expected noise power. This approach improves the performance because no power is spent on transmitting the unreliable bits, and all the available power can be dedicated to transmitting the more reliable bits.

We next describe the method of construction of $\tilde{r}$.

2) Construction of $\tilde{r}$: Consider the transmission model

$$\tilde{r} = \alpha_{1,3} x + \tilde{n}$$

where x is selected from signal constellation $\mathcal{A}_1 = \{c_1, c_2, \cdots, c_{r_1}\}$ and $\tilde{n}$ is zero mean AWGN noise $\tilde{n}$ with unknown variance σ_2^2 per complex dimension. Suppose that $p_i = p(x = c_i|\tilde{r})$, $i = 1, 2, \cdots, r_1$ are given, where $p_i \triangleq p\,(x = c_i|r)$. We are interested in finding the value of $\tilde{r}$ so that the expected power of error

$$\sum_{i=1}^{r_i} p_i |\tilde{r} - c_i|^2$$

is minimized.

Theorem 1. *The optimum point $\tilde{r}$ is given by*

$$\tilde{r} = p_1 c_1 + p_2 c_2 \quad (3)$$

for 2-elements constellations, and

$$\tilde{r} \simeq \sum_{i=1}^{r_1} p_i c_i \quad (4)$$

for constellations with more than 2 points.

Proof: For simplicity, let $d_i = |\tilde{r} - c_i|$, $i = 1, 2, \cdots, r_1$. Without loss of generality (by a possible re-labeling) assume that

$$p_1 \geq p_2 \geq p_3 \geq \cdots \geq p_{r_1}.$$

Clearly, from the Gaussian model assumed for noise $\tilde{n}$

$$\frac{p_1}{p_i} = \frac{\exp\left(-\frac{d_1^2}{\sigma_2^2}\right)}{\exp\left(-\frac{d_i^2}{\sigma_2^2}\right)},$$

for $i = 1, 2, \cdots, r_1$. Thus

$$\ln p_1 - \ln p_i + \frac{d_1^2}{\sigma_2^2} = \frac{d_i^2}{\sigma_2^2},$$

and

$$\frac{1}{\sigma_2^2} \sum_{i=1}^{r_i} p_i |\tilde{r} - c_i|^2 = \frac{d_i^2}{\sigma_2^2} + \ln p_1 - \sum_{i=1}^{r_1} p_i \ln p_i.$$

Let $H(\mathcal{P}) = -\sum_{i=1}^{r_1} p_i \ln p_i$ denote the natural entropy of a distribution on $\mathcal{A}$ with $\mathcal{P}(c_i) = p_i$. We thus have

$$\frac{1}{\sigma_2^2} \sum_{i=1}^{r_i} p_i |\tilde{r} - c_i|^2 = \frac{d_i^2}{\sigma_2^2} + \ln p_1 + H(\mathcal{P}).$$

By changing the role of p_1 and p_j in the above, we have

$$\frac{1}{\sigma_2^2} \sum_{i=1}^{r_i} p_i |\tilde{r} - c_i|^2 = \frac{d_i^2}{\sigma_2^2} + \ln p_1 - \sum_{i=1}^{r_1} p_i \ln p_i$$

for all $j = 1, 2, \cdots, r_1$.

In other words, for the optimum $\tilde{r}$, we have

$$\frac{d_j^2}{\sigma_2^2} + \ln p_j + H(\mathcal{P}) = K_o, \qquad (5)$$

for all $j = 1, 2, \cdots, r_1$, where K_o is a constant. We need to choose K_o and σ_2 such that it minimizes

$$\sum_{i=1}^{r_i} p_i |r - c_i|^2 = d_1^2 + \sigma_2^2 \left\{\ln p_1 + H(\mathcal{P})\right\}. \qquad (6)$$

For simplicity, let

$$a_j = \ln p_j + H(\mathcal{P}).$$

Then

$$\frac{d_j^2}{\sigma_2^2} + a_j = K_o. \qquad (7)$$

Solution For 2-Elements Constellations: We first solve the above optimization problem for the case that $\mathcal{A}_1$ has $r_1 = 2$ elements. Clearly, for a fixed σ_2, minimization of the objective function $d_1^2 + \sigma_2^2 \left\{\ln p_1 + H(\mathcal{P})\right\}$ is achieved by making d_1 smaller. By (5), by reducing the value of d_1, the value of d_2 also decreases. However by triangle inequality

$$d_1 + d_2 \geq |c_1 - c_2|.$$

Thus for a fixed σ_2, the values of d_1 and d_2 that minimize the objective function are solution to the equation

$$d_1 + d_2 = |c_1 - c_2|.$$

Let $d = |c_1 - c_2|$, then from (7), we have

$$d_2^2 - d_1^2 = \sigma_2^2 (a_1 - a_2).$$

Thus

$$d_2 - d_1 = \frac{\sigma_2^2 (a_1 - a_2)}{d_1 + d_2} = \frac{\sigma_2^2 (a_1 - a_2)}{d}.$$

We conclude that

$$\begin{aligned} d_1 &= \frac{d}{2} - \frac{\sigma_2^2 (a_1 - a_2)}{2d}, \\ d_2 &= \frac{d}{2} - \frac{\sigma_2^2 (a_1 - a_2)}{2d}. \end{aligned}$$

The objective function is now easily seen to be

$$d_1^2 + a_1 \sigma_2^2 = \left\{\frac{d}{2} - \frac{\sigma_2^2 (a_1 - a_2)}{2d}\right\}^2 + a_1 \sigma_2^2. \qquad (8)$$

Taking derivative of the right-hand side in (8) w.r.t. σ_2^2, we can now observe that the minimizing value of σ_2^2 for the objective function is given by

$$\sigma_2^2 = -\frac{a_1 + a_2}{(a_1 - a_2)^2} d^2 = \frac{p_1 - p_2}{\ln p_1 - \ln p_2} d^2, \qquad (9)$$

and the corresponding optimizing values for d_1 and d_2 are given by

$$\begin{aligned} d_1 &= \frac{a_1}{a_1 - a_2} d, \\ d_2 &= \frac{-a_2}{a_1 - a_2} d. \end{aligned}$$

Simplifying we observe that

$$d_1 = p_2 d, \qquad (10)$$

$$d_2 = p_1 d. \qquad (11)$$

This means that the optimum point $\tilde{r}$ is given by

$$\tilde{r} = p_1 c_1 + p_2 c_2. \qquad (12)$$

Solution for Constellations with More Than 2 Points: Here, for given p_i, $i = 1, 2, \cdots, r_1$, there is not necessarily any point $\tilde{r}$ such that $p_i = p\left(x = c_i | \tilde{r}\right)$ for all $i = 1, 2, \cdots, r_1$ hold. If these were to hold, then the equations

$$\ln p_i - \ln p_j = -\frac{d_j^2}{\sigma_2^2} + \frac{d_i^2}{\sigma_2^2},$$

must be satisfied for all $i \neq j$. Any of these equations correspond to a line perpendicular to the line segment between c_i and c_j. These lines do not necessarily all go through the same point $\tilde{r}$. If this happens, we will say that p_i, $i = 1, 2, \cdots, r_1$ is a geometrically consistent probability distribution function.

Since a perfect match is not possible, i.e., $p_i \triangleq p(x = c_i | r) \neq p(x = c_i | \tilde{r})$ for $i = 1, \cdots, r_1$ and $r_1 > 2$, we seek to construct a point $\tilde{r}$ that its a posteriori p.d.f is close in Kullback-Leibler sense to p_i, $i = 1, 2, \cdots, r_1$, and also reduces the expected power of error $\sum_{i=1}^{r_i} p_i |\tilde{r} - c_i|^2$.

With the above notation the a posteriori p.d.f induced by $\tilde{r}$

is

$$Q(c_i) = p(x = c_i|\tilde{r}) = \frac{\exp\left(-\frac{d_i^2}{\sigma_2^2}\right)}{\sum_{j=1}^{r_i} \exp\left(-\frac{d_j^2}{\sigma_2^2}\right)},$$

and the Kullback-Leibler distance of distribution $\mathcal{P}$ and the above a posteriori distribution is easily computed to be

$$D(\mathcal{P}||\mathcal{Q}) = -H(\mathcal{P}) + \frac{1}{\sigma_2^2}\sum_{j=1}^{r_i} p_j d_j^2 + \ln \sum_{j=1}^{r_i} \exp\left(-\frac{d_j^2}{\sigma_2^2}\right).$$

The term $\sum_{j=1}^{r_i} p_j d_j^2$ is the expected power of error. At high signal to noise ratios, we can make the approximation by the dominant term

$$\ln \sum_{j=1}^{r_i} \exp\left(-\frac{d_j^2}{\sigma_2^2}\right) \simeq \frac{-\min_j d_j^2}{\sigma_2^2} = \frac{-d_1^2}{\sigma_2^2}.$$

Thus

$$D(\mathcal{P}||\mathcal{Q}) \simeq -H(\mathcal{P}) + \frac{1}{\sigma_2^2}\left\{\left(\sum_{j=1}^{r_i} p_j d_j^2\right) - d_1^2\right\}.$$

In high signal to noise ratios, we expect that only the correct point and one of the nearest neighbors have non-negligible a posteriori probabilities p_1 and p_2. In this case, we know from the above analysis that $d_1^2 \simeq p_2^2 d_2$. Thus we observe that minimization of the Kullback-Leibler distance is approximately equivalent to minimization of the expected power of error $\sum_{j=1}^{r_i} p_j d_j^2$. This is easily seen to be accomplished by choosing

$$\tilde{r} = \sum_{i=1}^{r_1} p_i c_i,$$

which proves the theorem. ■

Note that, in low signal to noise ratios, it is possible that the approximation in (4) is not close to the optimum point $\tilde{r}$. In this case, we can find the optimum point $\tilde{r}$ numerically by an iterative method for optimization: $\min_{\tilde{r},\sigma_2} D(\mathcal{P}||\mathcal{Q}(c_i))$ and $\min_{\tilde{r},\sigma_2} \sum_{i=1}^{r_1} p_i(\tilde{r} - c_i)^2$.

IV. Robust Scalable Physical Layer Network Coding Scheme

Although the core of above approach must be now clear as we have outlined the construction of $\tilde{r}$ in Section III-B2, we still have to address the following questions:

- How would the destination node determine the unreliable bits eliminated at each intermediate transmission stage? What kind of side information must be presented to the receiver?
- What if each node uses higher order constellations?
- The strategy may change from one symbol transmission at v_3 to the other depending on the noise and potential channel variations. This may produce a requirement of presence of a large amount of side information to be present at each intermediate node. How can this be avoided?
- If in the above the bit b_2 has low reliability and we eliminate it at an intermediate node, it may not be possible to recover this information at the destination node. Is the elimination of this bit the right strategy?

We next address these issues in detail and present our full approach to the proposed physical layer network coding scheme.

A. Details of Our Approach

We now describe the details of our physical layer networking approach. First, we consider a network where multiple transmitting nodes transmit to a single node. Then we extend the method to transmission of arbitrary nodes (relay or source nodes) to arbitrary nodes.

1) Transmission Strategy for Higher Order Constellations with Multiple Simultaneous Transmitting Sources: To this end, let the signal constellation employed at node $v_i = S_i$, $i = 1, 2, \cdots, k$ be respectively $\mathcal{A}_i$, $i = 1, 2, \cdots, k$ respectively r_i, $i = 1, 2, \cdots, k$ elements. Node v_j receives source packet $P_{i,j}$ from node v_i, $i = 1, 2, \cdot, k$ with average power P per transmitting node. Let the packet $P_{i,j}$ be encoded by l constellation symbols

$$X_{i,j}^1 X_{i,j}^2 \cdots X_{i,j}^l$$

for $i = 1, 2, \cdot, k$ and assume that the received word at v_j be given by

$$Y_j^1 Y_j^2 \cdots Y_j^l.$$

The received signal Y_j^m, $m = 1, 2, \cdots, l$ is modeled as the corrupted version of the linear combination of scaled versions of $X_{i,j}^m$ by n_j^m, i.i.d. samples of complex additive noise n_j with mean zero and variance $\sigma^2/2$ per real dimension. In mathematical notation

$$Y_j^m = \sum_{i=1}^{k} \alpha_{i,j} X_{i,j}^m + n_j^m,$$

for $m = 1, 2, \cdots, l$, where $\alpha_{i,j}$ denotes the channel from node v_i, $i = 1, 2, \cdots, k$ to the node v_j.

Since we are dealing with potentially coded signals, the receiver v_j does symbol by symbol signal transformation to reduce complexity. This means that it treats the symbols $X_{i,j}^1 X_{i,j}^2 \cdots X_{i,j}^l$ for $i = 1, 2, \cdots, k$ as uncoded symbols and computes the LLRs of symbols $X_{i,j}^m$, $i = 1, 2, \cdots, k$ from Y_j^m for $m = 1, 2, \cdots, l$.

For each $i = 1, 2, \cdots, k$, $m = 1, 2, \cdots, l$, and $c \in \mathcal{A}_i$, we can compute the log-likelihood

$$\mathrm{LLR}_{i,m}(c) = \ln \frac{p\left(X_{i,j}^m = c|r\right)}{p\left(X_{i,j}^m \neq c|r\right)}.$$

Let

$$\mathrm{LLR}\left(X_{i,j}^m\right) = \max_{c \in \mathcal{A}_i} \mathrm{LLR}_{i,m}(c),$$

for $i = 1, 2, \cdots, k$ and $m = 1, 2, \cdots, l$. The node v_j then computes the average function

$$\mathrm{LLR}_i = \mathrm{LLR}\left(X_{i,j}^1 X_{i,j}^2 \cdots X_{i,j}^l\right) = \frac{\sum_{m=1}^{l} \mathrm{LLR}\left(X_{i,j}^m\right)}{l},$$

for $i = 1, 2, \cdots, k$. Other functions are possible. However, it can be easily seen that for the transmission of uncoded signals, the above LLRs are closely related to the average virtual signal to noise ratio of each symbol stream.

We further assume that the node v_j has selected thresholds T_j^i, $i = 1, 2, \cdots, k$ that respectively indicates the LLR quality of data streams from v_i, $i = 1, 2, \cdots, k$ suitable for decoding. The node v_j chooses all nodes that satisfy $\text{LLR}_i > T_j^i$. If there are no such nodes then v_j determines that neither of streams $X_{i,j}^1 X_{i,j}^2 \cdots X_{i,j}^l$, $i = 1, \cdots, k$ have good quality. Any further amplification and re-transmission can only decrease the quality of these underlying streams in the transmitted signal. Thus, the node v_j refrains from transmission and saves its energy.

Thus we can assume without loss of generality (by re-labeling) that for some $1 \leq k_1 \leq k$, $\text{LLR}_i > T_j^i$ for $i = 1, 2, \cdots, k_1$ and $\text{LLR}_i \leq T_j^i$ for $k_1 < i \leq k$.

Consider

$$\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_{k_1}$$

and all the $r_1 r_2 \cdots r_{k_1}$ linear sums

$$\mathcal{A}\left(\alpha_{1,j}, \cdots, \alpha_{k_1,j}\right) = \left\{ \sum_{i=1}^{k_1} \alpha_{i,j} a_i \middle| a_i \in \mathcal{A}_i \right\}.$$

For each $m = 1, 2, \cdots, l$, the node v_j finds 3 elements $c_{1,m}$, $c_{2,m}$ and $c_{3,m} \in \mathcal{A}\left(\alpha_{1,j}, \cdots, \alpha_{k_1,j}\right)$ that maximize the

$$\text{LLR}_{i,m}(c) = \ln \frac{p\left(\sum_{i=1}^{k_1} \alpha_{i,j} X_{i,j}^m = c \middle| r\right)}{p\left(\sum_{i=1}^{k_1} \alpha_{i,j} X_{i,j}^m \neq c \middle| r\right)}$$

over all $c \in \mathcal{A}\left(\alpha_{1,j}, \cdots, \alpha_{k_1,j}\right)$. Note that 3 is chosen by convenience as a reasonably small number.

The node v_j next constructs a new received word $\tilde{Y}_j^m$ for the transmission model $\tilde{Y}_j^m = X + \tilde{n}$, where $X \in \mathcal{A}$ is the transmitted signal and $\tilde{n}$ is complex Gaussian noise with mean zero and variance σ_2^2 per complex dimension such that $p(X_j^m | \tilde{Y}_j^m)$ is almost the same as in Kullback-Leibler distance sense $p\left(X_j^m | Y_j^m\right)$ and also minimizes the expected power of the error for all $m = 1, 2, \cdots, l$. The construction of $\tilde{Y}_j^m$ has been outlined in Section III-B2. The node v_j then scales $\tilde{Y}_j^1 \tilde{Y}_j^2 \cdots \tilde{Y}_j^l$ by a constant factor β such that the sequence $\beta \tilde{Y}_j^m$, $m = 1, 2, \cdots, l$ has average power P (or satisfies any other desired power constraint e.g. peak power). It then transmits this scaled sequence.

2) Extension to Transmission by Arbitrary Nodes: We now treat the case where the transmitting nodes v_i, $i = 1, 2, \cdots, k$ are not necessarily source nodes. Our transmission scheme described below inductively guarantees that the signal transmitted at each node is a linear sum of source signals perturbed by noise. This is clearly true for nodes in layer zero of the network (source nodes). The construction presented in the above guarantees that this is also true for the nodes in layer one. Inductively assume that nodes $v_i, i = 1, 2, \cdots, k$ in layer $q \geq 1$ are transmitting signals that are linear combinations of the source node message perturbed by noise. Let the support of signal transmitted by v_i, denoted by $\text{supp}(v_i)$, be the set of all source nodes $S_1, S_2, \cdots, S_N$ that appear in the linear summation part of the signal transmitted by node v_i. We let

$$\text{supp}(v_i,\ i = 1, 2, \cdots, k) = \cup_{i=1}^k \text{supp}(v_i).$$

Clearly the signal at node v_j is a linear sum of the signals of the sources that only appear in $\text{supp}(v_i,\ i = 1, 2, \cdots, k)$. This means that as far as node v_j is concerned, the scenario is analogous to the case that sources in $\cup_{i=1}^k \text{supp}(v_i)$ transmit directly to v_j with appropriately selected channel coefficients. Thus the node v_j can apply the same network coding strategy as the one described in Section IV-A1. It is clear that in this way, the transmitted signal by v_j is also a linear combination of signals transmitted by sources perturbed by noise. This completes the induction step. Thus the presentation of our network coding scheme is complete.

3) The Network Coding Header: We will now discuss the headers required for the above network coding scheme. Clearly, for the above scheme to work it suffices that the header of each transmitted signal by a node v_j contains information about its support, $\text{supp}(v_j)$. In particular, the header may only consists of the index of source nodes in the support of the transmitted signal.

It must be noted that since we will need to estimate the effective channel coefficients at each receiver node, we may want to include known pilot sequences present at each transmitted packet from each source node. We do not count this as the header since these pilot sequences may have to exist even in a classical scenario where no network coding is employed.

4) Derivation of the Receiver: We next discuss the receiver structure at each destination node. Each destination node $T_j,\ j = 1, 2, \cdots, M$, depending on the transmission strategy in the prior layer receives a number M_j of linear summations of the transmitted source packets from $N_j \leq N$ of sources $S_1, \cdots, S_N$ perturbed by noise. This is analogous to a wireless multiuser transmission (multi-access channel) scenario with N_j transmitter where the receiver has M_j receive antennas. It could also thought of as an $N_j \times M_j$ MIMO transmission scenario, where the transmit antennas send independent coded signals. Thus various MIMO receivers (e.g. the BLAST receiver, full maximum likelihood receiver, etc.) can be applied at T_j in order to decode the source packets. For instance if $M_j = N_j$, a MIMO channel inversion can be applied to separate the transmitted signals by various sources. This strategy is simple but obviously non-optimal.

B. Further Enhanced Robustness and Scalability

It is clear that our proposed scheme is both robust and scalable. However, further granularity can be achieved by choosing constellations at each source that lend itself to a multilevel structure. For example, the QPSK constellation can be thought of as a scaled sum of a BPSK constellation with another BPSK constellation shifted by j, i.e.

$$\text{QPSK} = \frac{\sqrt{2}}{2}\text{BPSK} + j\frac{\sqrt{2}}{2}\text{BPSK}$$

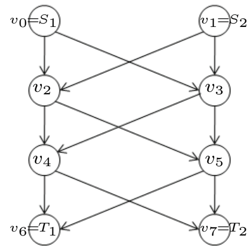

Fig. 1. 4-layer wireless network with two source nodes S_1, S_2, four intermediate nodes, and two destination nodes T_1, T_2.

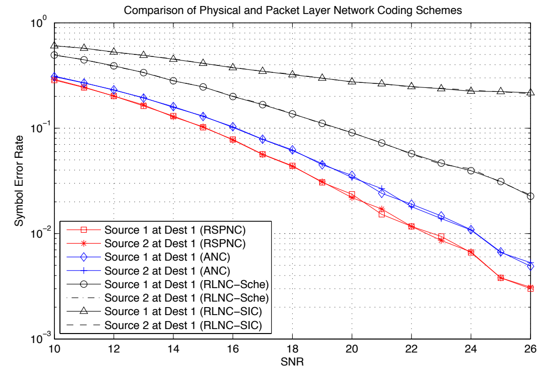

Fig. 2. Performance comparison of algebraic network coding schemes (RLNC-Sche, RLNC-SIC), analog network coding (ANC) and the new robust scalable physical layer network coding scheme (RSPNC) at destination node 1 for uncoded QPSK transmission with $\sigma_2^2 = \sigma_3^2 = 2\sigma_4^2 = 2\sigma_5^2 = 2\sigma_6^2 = 2\sigma_7^2$.

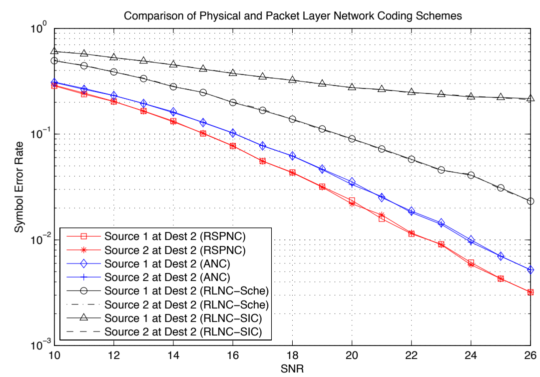

Fig. 3. Performance comparison of algebraic network coding schemes (RLNC-Sche, RLNC-SIC), analog network coding (ANC) and the new robust scalable physical layer network coding scheme (RSPNC) at destination node 2 for uncoded QPSK transmission with $\sigma_2^2 = \sigma_3^2 = 2\sigma_4^2 = 2\sigma_5^2 = 2\sigma_6^2 = 2\sigma_7^2$.

and the 16-QAM constellation can be written as

$$\text{16-QAM} = \frac{2}{\sqrt{5}}\text{QPSK} + j\frac{1}{\sqrt{5}}\text{QPSK}.$$

Similarly the 32-QAM and 64-QAM can be written as sums of scaled versions of BPSK and QPSK constellations.

A source S_i transmitting using, for instance, the 16-QAM constellation can be then considered as sum of virtual sources S_i^1 and S_i^2 transmitting QPSK symbols with channel gains $\frac{2}{\sqrt{5}}$ and $\frac{1}{\sqrt{5}}$. Based on our method each node can then transmits signals of only one of these virtual sources when the other is not reliable. Clearly, this provides further robustness to poor channel conditions.

V. Numerical Analysis

In this section, we provide simulation results and document the performance of our scheme. We consider the scenario depicted in Figure 1. Here the network consists of two sources $v_0 = S_1$ and $v_1 = S_2$ in layer 0, two relay nodes v_2 and v_3 in layer 1, two relay nodes v_4 and v_5 in layer 2 and two destination nodes $v_6 = T_1$ and $v_7 = T_2$ in layer 3. The channel gains $\alpha_{i,j}$ between node i of each layer $k = 0, 1, 2$ and node j of layer $k + 1$ is modeled by i.i.d samples of a circularly symmetric complex Gaussian $\mathcal{N}(0, 1)$ with variance 0.5 per real dimension for all i and j. The received signal at each node v_j is assumed to be corrupted by i.i.d. samples of a circularly symmetric complex Gaussian noise $\mathcal{N}\left(0, \sigma_j^2\right)$ with variance $\sigma_j^2/2$ per real dimension. It is assumed that the average transmit power for each transmitting node is 1. The value SNR is thus defined to be $\text{SNR} = \frac{1}{\sigma_2^2}$.

For all schemes except one of algebraic network coding schemes, referred to RLNC-Sche, it is assumed that transmissions of nodes $v_0 = S_1$ and $v_1 = S_2$ are received simultaneously at relay nodes v_2 and v_3. Likewise transmissions from v_2 and v_3 are received simultaneously at relay nodes v_4 and v_5. Followed by that the transmissions of nodes v_4 and v_5 are separated in time and are received separately at destination nodes $v_6 = T_1$ and $v_7 = T_2$. We assume that the source nodes transmit using uncoded QPSK. For simplicity all the thresholds are set to zeros. At each destination node maximum likelihood (ML) decoding is performed. As for RLNC-Sche, the transmissions of all nodes are separated in time and are received separately at relay or destination nodes by scheduling. This scenario is more favorable to algebraic network coding than the scenario where simultaneous transmissions and receptions occur as described earlier, because there is no interference at each relay node, when it decodes before applying random linear network coding at the algebraic-level.

In Figures 2 and 3, we provide simulation results for $\sigma_2^2 = \sigma_3^2 = 2\sigma_4^2 = 2\sigma_5^2 = 2\sigma_6^2 = 2\sigma_7^2$. Figures 2 and 3 respectively depict the performance for the signal received at Destinations 1 and 2 for algebraic network coding schemes (RLNC-Sche, RLNC-SIC), analog network coding (ANC) and the new robust scalable physical layer network coding scheme (RSPNC). As can be seen, the new RSPNC scheme provides coding gains of about 2 [dB], $4 - 6$ [dB], and $10 - 14$ [dB] more than ANC, RLNC-Sche, and RLNC-SIC, respectively. Also as expected the performance at two destination points are identical since the channel conditions are symmetric, and RLNC-SIC performs poorly, because of strong interferences over the overhearing channels. Additionally, in Figures 4 and

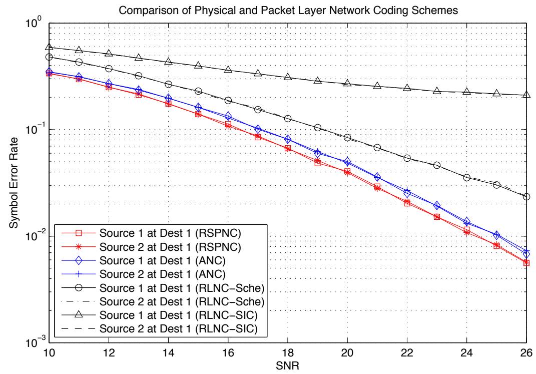

Fig. 4. Performance comparison of algebraic network coding schemes (RLNC-Sche, RLNC-SIC), analog network coding (ANC) and the new robust scalable physical layer network coding scheme (RSPNC) at destination node 1 for uncoded QPSK transmission with $2\sigma_2^2 = 2\sigma_3^2 = \sigma_4^2 = \sigma_5^2 = \sigma_6^2 = \sigma_7^2$.

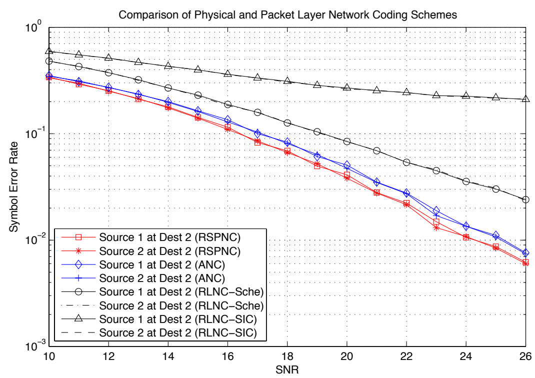

Fig. 5. Performance comparison of algebraic network coding schemes (RLNC-Sche, RLNC-SIC), analog network coding (ANC) and the new robust scalable physical layer network coding scheme (RSPNC) at destination node 2 for uncoded QPSK transmission with $2\sigma_2^2 = 2\sigma_3^2 = \sigma_4^2 = \sigma_5^2 = \sigma_6^2 = \sigma_7^2$.

5, we provide performance results for $2\sigma_2^2 = 2\sigma_3^2 = \sigma_4^2 = \sigma_5^2 = \sigma_6^2 = \sigma_7^2$. Here, there is less noise to remove in the first stage so the coding gains are about 1 [dB], $3-4$ [dB], and $7-13$ [dB] more than ANC, RLNC-Sche, and RLNC-SIC, respectively. Based on these figures, it is observed that compared to the classical physical layer network coding schemes, the new RSPNC scheme provides performance improvements in addition to added scalability and locality.

VI. Concluding Remarks

This paper investigates robust scalable physical layer network coding (RSPNC), which transforms the received signal at a relay node before re-transmission by eliminating the unreliable components of the received signal, and thus improves the reliability of physical layer network coding. The new RSPNC scheme is also naturally amenable to scalable and distributed implementation. Numerical results show that in 4-layer wireless network the new RSPNC scheme provides coding gains of $1-2$ [dB], $4-6$ [dB], and $7-14$ [dB] more than analog network coding, random linear network coding with scheduling, and random linear network coding without scheduling, respectively. By changing noise variances, we also show that the new RSPNC scheme performs more reliably than classical physical network coding schemes in a wireless network where there is more noise to remove.

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