Successive Cancellation Integer Forcing via Practical Binary Codes

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***Abstract*—A new multiple-input multiple-output (MIMO) receiver scheme for practical binary codes is proposed that provides consistent gains over conventional linear receivers. We first develop a practical successive integer forcing (IF) scheme based on practical binary codes rather than lattice codes. We then present the successive cancellation integer forcing (SC-IF) scheme, which combines and enhances successive IF and minimum mean squared error successive interference cancellation (MMSE-SIC). In this scheme, the receiver first decides whether individual decoding or IF sum decoding is appropriate for each data stream, and then conducts successive IF sum decoding only for selected streams while decoding the remaining streams using MMSE-SIC. The proposed SC-IF methodology mitigates the performance loss caused by mismatched IF filtering in fading channels, while attenuating the noise amplification caused by MMSE filtering. Extensive link-level simulations demonstrate that the proposed successive IF significantly improves the basic IF, and the SC-IF improves both the successive IF and MMSE-SIC, offering uniform improvements over conventional linear receivers for most channel correlation and variation parameters and modulation orders at comparable computational costs. These results illustrate the viability of SC-IF as a fundamental building block for high-performance MIMO receivers in 5G-Advanced and/or subsequent-generation communication systems.**

***Index Terms*—Binary code, integer forcing, linear receiver, multilevel coding (MLC), multiple-input multiple-output (MIMO), successive interference cancellation (SIC)**

I. Introduction

In recent years, the volume of mobile data traffic has increased dramatically. In particular, global mobile data traffic has reached 51 Exabytes (EB) per month by the end of 2020, and is projected to increase by a factor of 4.5 to 226 EB per month by 2026 [1]. The fourth-generation cellular system cannot fully support emerging new services that are driving this trend, such as 4K streaming and virtual/augmented reality from interactive live concerts and sporting events for immersive experiences on mobile devices. To fulfill such enhanced mobile broadband (eMBB) usage scenarios, the fifth generation (5G) system would be required to produce three times more spectral efficiency than its predecessor [2]. This is anticipated to be done by a more efficient utilization of wireless spectrum. As a result, it is imperative to build a low-complexity multiple-input multiple-output (MIMO) receiver capable of delivering high performance while scaling with many antennas and large constellations.

In the literature, there are two types of MIMO receivers: nonlinear and linear. A nonlinear receiver, such as a joint maximal likelihood (ML) detection receiver, processes the signals detected at several receive antennas jointly. This technique provides the best detection performance at a cost of computation that grows exponentially with the product of the number of transmit antennas and the cardinality of the modulation constellation. In contrast, linear receivers, such as a zero-forcing (ZF) receiver (or decorrelator) and a minimum mean squared error (MMSE) receiver, decouple the signals from multiple transmit antennas by applying a linear filter for channel inversion and then recover the data streams independently. The complexity is greatly decreased compared to nonlinear receivers. The primary disadvantage of the ZF receiver is the high noise amplification caused by channel inversion. By inverting the channel using a regularized channel inversion matrix that maximizes the signal-to-noise ratio (SNR) of individual signals, the MMSE receiver mitigates the performance decrease caused by the noise amplification problem. Compared to ZF receivers, MMSE receivers perform especially well when the SNR is low. The performance of the MMSE receiver can be further improved by combining it with successive interference cancellation (SIC). Symbol-level and codeword-level SIC can both boost the effective SNR, resulting in a performance advantage over MMSE receivers. Due to its low complexity, linear MIMO receivers such as ZF, MMSE, and MMSE-SIC have been frequently utilized in practice [3]. As demonstrated in [4], the decoupling procedure by linear filtering still incurs substantial noise amplification, even for the MMSE-SIC receiver, when the channel matrix is almost singular; for the complete analysis, see [3], [5]–[7] and the references therein.

LR-aided MIMO receivers have been proposed to reduce

Manuscript received Oct. 12, 2021; revised May 09, 2022, Oct. 03, 2022, and Dec. 06, 2022; accepted Jan. 27, 2022. The work of S.-K. Ahn was supported by Institute of Information & communications Technology Planning & Evaluation (IITP) grant funded by the Korea government (MSIT) (No. 2020-0-00846, Development of Convergence Transmission Technology for 5G and ATSC 3.0 Networks). The work of S. H. Chae was supported by Institute of Information & communications Technology Planning & Evaluation (IITP) grant funded by the Korea government (MSIT) (No.2021-0-00746, Development of Tbps Wireless Communication Technology) and the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) under Grant 2021R1F1A1062539. The work of K. T. Kim was supported by IEDC grant A293-9-21Fund-10245. *(Corresponding author: Sung Ho Chae.)*

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the performance deterioration caused by noise amplification in conventional linear receivers [8], [9]. The LR-aided MIMO receiver changes the basis vectors of the channel matrix into reduced basis vectors, therefore decreasing the channel matrix's condition number. Specifically, it initially estimates the integer combination of the transmitted symbols in relation to the basis changed by LR. The inverse transform is then applied to the estimated symbols to recover the transmitted symbols. In correlated fading conditions, the LR-aided MIMO receiver offers superior performance improvements over ZF or MMSE receivers. However, when combined with forward error correction codes, it is not simple for the LR-assisted MIMO receiver to create soft outputs on the transmitted symbols, as only one symbol vector is estimated. Several list-based soft demodulators have been proposed to alleviate this problem [10], [11]. Nonetheless, these additional procedures significantly increase its computing complexity, particularly for handling a large quantity of data streams, high modulation orders, or both.

More recently, a novel linear MIMO receiver technique, *integer forcing* (IF), has been introduced to address the previously described noise amplification issue of linear receivers at low complexity [12]. In IF, each data stream is encoded with the same lattice code, and a linear filter is used to form an effective channel with integer coefficients. In contrast to typical linear receivers, each single-input single-output (SISO) decoder tries to decode integer-linear combinations of transmitted codewords rather than individual codewords. Due to the lattice condition that these linear combinations are themselves codewords, this decoding procedure is feasible. Once the linear combinations are recovered, it is simple to retrieve the original messages from them. Any set of independent linear combinations (typically regarded as a full-rank integer matrix) may be employed in IF, which offers an additional degree of freedom to maximize the effective SNR and minimize noise amplification. In comparison, the effective channel matrices of conventional linear receivers such as ZF and MMSE are restricted to the identity matrix since they are designed to decouple the codewords. It has been demonstrated that, under idealized lattice codes, IF receivers can outperform traditional linear receivers at almost the same complexity, achieving the optimal diversity-multiplexing tradeoff [12]. Similar to LR, IF receivers initially estimate the integer combination of transmitted symbols in relation to the transformed basis derived from an IF filter. In the second stage, however, unlike LR, IF decodes integer-linear combinations of transmitted codewords before applying the inverse transform to the estimated symbols in the signal space. IF resolves integer interference via $\mathbf{A}^{-1}$ over $\mathbb{Z}_2$, not in the signal space, i.e. over the complex/real numbers, where $\mathbf{A}$ is a full-rank integer matrix.

Several studies have been conducted to enhance the performance of the basic IF scheme. It has been shown in [13] that the capacity within a constant gap for general MIMO channels can be achieved by applying precoding with the generating matrix of a perfect linear dispersion space-time code on the transmitter side and IF equalization on the receiver side, under the assumption that the transmitter has no channel state information (CSI) and only white-input mutual information. In addition, *successive IF*, which combines IF sum decoding with SIC operations, has been proposed in [14], [15]. This technique aims to decode integer-linear combinations of codewords one-by-one in sequence, as opposed to the basic IF receiver, which recovers them in parallel. The slowest descent method [16] and the Hermite-Korkine-Zolotareff (HKZ) and Minkowski lattice basis reduction algorithms [17] are examples of low-complexity algorithms that have been researched for locating a suitable integer matrix for IF. Recently, a channel-variation-resistant integer matrix search algorithm has been proposed in [18]. In addition, the precise performance characterization of two parallel channels using precoding with the full-diversity rotation matrix at the transmitter and IF equalization at the receiver has been investigated [19]. Combining IF and extended spatial modulation, spatially modulated IF (SM-IF) has been designed and implemented with practical binary codes in [20]. Multi-mode IF (M-IF) using various integer-valued effective channel matrices for IF sum decoding has been proposed more recently for block fading channels [21]. In addition, multiple-access channels [15], [22], broadcast channels [22]–[25], interference channels [26]–[30], and relay networks [31], [32] are examples of research that have recently been published that extend the basic IF scheme to accommodate multi-user communication.

Due to their complexity, however, lattice codes used in IF schemes [12]–[15] are rather difficult to implement in practice. A simple q-ary implementation of compute-and-forward has been proposed in [33], and IF receivers with binary codes (i.e., $q = 2$) that use practical off-the-shelf codes such as long-term evolution (LTE) turbo or 5G new radio (NR) low-density parity-check (LDPC) codes instead of lattice codes have also been proposed [4], [34]. The basic coding approach described in [34] encodes the least significant bits (LSBs) of a modulated symbol with binary codes and applies IF receivers to these coded bits, leaving the remaining bits uncoded. A more contemporary technique described in [4] makes full use of binary codes by multilevel encoding of binary linear codes at the transmitter and multistage decoding suited to the IF equalization. Section II reviews the basic IF scheme and the MMSE-SIC scheme for quadrature phase-shift keying (QPSK) modulation and practical binary linear codes.

A. Our Contribution

Our primary goal is to design a practical MIMO receiver scheme based on commercially available binary codes that provides consistent gains over conventional linear receivers in a variety of real-world settings. The following is a summary of the main contributions of this paper:

- We introduce the first practical coding scheme for successive IF, one that uses commonly available binary codes rather than theoretical lattice codes. It is demonstrated that the proposed successive IF scheme can significantly improve the basic IF scheme not only in theory, but also when implemented using off-the-shelf binary codes.
- We further design a successive cancellation IF (SC-IF) scheme that merges successive IF and MMSE-SIC into a single MIMO framework in order to efficiently

harness the benefits of each technique. Therefore, the SC-IF always outperforms successive IF and MMSE-SIC with practical binary codes across the whole SNR range, assuming there is no noise estimation error. Additionally, for most channel conditions we study, the SC-IF still beats successive IF and MMSE-SIC, otherwise assuring at least approximately the same performance, even in the presence of noise estimation error.
- Both the proposed successive IF scheme and the SC-IF scheme are capable of handling any modulation order with any practical binary codes thanks to the use of multilevel coding (MLC) and natural mapping [4].

The key idea of the proposed coding scheme for successive IF is to estimate the effective noise as precisely as feasible in practical environments where the channel can change over the codeword length when employing binary codes. Additionally, the key concept of the SC-IF is that the receiver executes successive IF sum decoding only for selected data streams, as opposed to all data streams, while the remaining data streams are individually decoded by MMSE-SIC, with MMSE-SIC individual decoding occurring prior to successive IF sum decoding. Taking into account that MMSE-SIC can improve the channel quality not only for the remaining individual codewords but also for the sum of remaining codewords for IF sum decoding, the proposed SC-IF scheme outperforms both successive IF and MMSE-SIC, uniformly across the majority of channel correlation and variation parameters.

B. Organization and Notation

The rest of the paper is organized as follows. In Section II, we describe the system model and review the basic IF scheme in [4] and the codeword-level MMSE-SIC scheme. In Section III, we present the proposed coding scheme for successive IF and then the SC-IF scheme built on practical binary codes. In Section IV, we describe how to extend the proposed schemes to higher modulation in detail. In Section V, extensive numerical simulations are performed to demonstrate the performance gains of the proposed approaches over conventional MIMO receivers in various settings. Section VI concludes the paper.

Throughout the paper, boldface lowercase letters are used to denote column vectors, and boldface uppercase letters are used to denote matrices. For a matrix $\mathbf{A}$, let $\mathbf{A}^\dagger$ and $\det(\mathbf{A})$ denote the transpose and the determinant of $\mathbf{A}$, respectively. The notation $\mathbf{I}_n$ and $\mathbf{1}_{n\times 1}$ denote the $n\times n$ identity matrix and the $n\times 1$ all-one vector, respectively. Let $[1:n]=\{1,2,\cdots,n\}$. The real and imaginary parts of a matrix $\mathbf{A}$ are denoted by $\mathrm{Re}(\mathbf{A})$ and $\mathrm{Im}(\mathbf{A})$, respectively. For a real number x, $x \mod 2$ denotes the output of the modulo-2 operation on x, where $x \mod 2$ is in $[0:2)$. Calligraphic letters $\mathcal{X}$, $\mathcal{Y}$, … are used to denote finite sets. We use $a := b$ to denote that a is equal to b by definition. We denote the circularly symmetric complex Gaussian distribution with mean 0 and variance σ^2 by $\mathcal{CN}(0,\sigma^2)$.

II. Preliminaries

A. System Model

Consider a MIMO 2^L-quadrature amplitude modulation (QAM) transmission with M transmit antennas and N receive antennas, where L is assumed to be multiple of two and $N \geq M$. Let $\mathcal{X}_L$ denote the set of 2^L-QAM constellation points. The channel output at resource element t is given by

$$\mathbf{y}(t)=\mathbf{H}(t)\mathbf{x}(t)+\mathbf{z}(t), \tag{1}$$

where $\mathbf{x}(t)=[\begin{array}{cccc} x_1(t) & x_2(t) & \cdots & x_M(t)\end{array}]^\dagger \in \mathbb{C}^{M\times 1}$ is the complex-valued input vector of the transmitter where $x_i(t)\in\mathcal{X}_L$, $\mathbf{H}(t)\in\mathbb{C}^{N\times M}$ is the complex-valued channel matrix between the transmitter and the receiver, and $\mathbf{z}(t)\in\mathbb{C}^{N\times 1}$ is the complex noise vector at the receiver with $\mathbf{z}(t)\sim\mathcal{CN}(\mathbf{0}_{N\times 1}, 2\mathbf{I}_N)$. It is assumed that $\mathbf{z}(t)$ is independent of the input vector and also over resource elements. In addition, the coefficients in $\mathbf{H}(t)$ are assumed to be known to the receiver but unknown to the transmitter. Note that we can equivalently rewrite (1) in the form of a real-valued representation as

$$\underbrace{\begin{bmatrix}\mathrm{Re}(\mathbf{y}(t))\\ \mathrm{Im}(\mathbf{y}(t))\end{bmatrix}}_{\bar{\mathbf{y}}(t)}=\underbrace{\begin{bmatrix}\mathrm{Re}(\mathbf{H}(t)) & -\mathrm{Im}(\mathbf{H}(t))\\ \mathrm{Im}(\mathbf{H}(t)) & \mathrm{Re}(\mathbf{H}(t))\end{bmatrix}}_{\bar{\mathbf{H}}(t)}\underbrace{\begin{bmatrix}\mathrm{Re}(\mathbf{x}(t))\\ \mathrm{Im}(\mathbf{x}(t))\end{bmatrix}}_{\bar{\mathbf{x}}(t)} + \underbrace{\begin{bmatrix}\mathrm{Re}(\mathbf{z}(t))\\ \mathrm{Im}(\mathbf{z}(t))\end{bmatrix}}_{\bar{\mathbf{z}}(t)}. \tag{2}$$

Hereafter, we consider the $2N\times 2M$ real-valued MIMO channel in (2).

We assume that communication takes place over n resource elements. Each transmit antenna $i\in[1:2M]$ attempts to send an independent binary data stream $\mathbf{w}_i$ with the average power constraint P, i.e., $\frac{1}{n}\sum_{t=1}^{n}|\bar{x}_i(t)|^2\leq P$ for all $i\in[1:2M]$, where $\bar{x}_i(t)$ is the i-th element in $\bar{\mathbf{x}}(t)$. In Sections II and III, we focus on the simplest case where $L=2$, i.e., QPSK for ease of explanation. The extension to higher-order modulation will be demonstrated in Section IV.

B. Review of the Previous IF Scheme in [4]

We first briefly review the recently developed IF scheme in [4], which can be implemented with practical binary linear codes.

1) Transmitter side: Recall that each transmit antenna i attempts to send an independent binary data stream $\mathbf{w}_i\in\mathbb{Z}_2^{\kappa\times 1}$ for $i\in[1:2M]$. To communicate $\mathbf{w}_i$, the channel encoder at the i-th transmit antenna uses a binary channel code to form a length-n binary linear codeword $\mathbf{b}_i\in\mathbb{Z}_2^{n\times 1}$ with code rate $r=\frac{\kappa}{n}$. Assuming that each encoder employs the same linear code, each transmit antenna i sends a modulated symbol $\bar{x}_i(t)\in\mathrm{Re}(\mathcal{X}_2)$ at resource element t as

$$\bar{x}_i(t)=\alpha\left(b_i(t)-\beta\right),$$

where $b_i(t)$ is the t-th bit in $\mathbf{b}_i$, $\alpha=2\sqrt{P}$, and $\beta=\frac{1}{2}$. Here, α is set to satisfy the transmit power constraint and β is chosen such that the average power of the constellation points is minimized.

2) Receiver side: Upon observing $\bar{\mathbf{y}}(t)$, the receiver applies a linear filter $\mathbf{F}_{\mathrm{IF}}(t)\in\mathbb{R}^{2M\times 2N}$ to get

$$\begin{aligned}\tilde{\mathbf{y}}(t)&=\mathbf{F}_{\mathrm{IF}}(t)\bar{\mathbf{y}}(t)\\ &=\mathbf{F}_{\mathrm{IF}}(t)\bar{\mathbf{H}}(t)\bar{\mathbf{x}}(t)+\mathbf{F}_{\mathrm{IF}}(t)\bar{\mathbf{z}}(t)\end{aligned}$$

$$= \mathbf{A}\bar{\mathbf{x}}(t) + \underbrace{\left((\mathbf{F}_{\text{IF}}(t)\bar{\mathbf{H}}(t) - \mathbf{A})\bar{\mathbf{x}}(t) + \mathbf{F}_{\text{IF}}(t)\bar{\mathbf{z}}(t)\right)}_{\text{Effective noise}} \quad (3)$$

where

$$\begin{aligned}\mathbf{F}_{\text{IF}}(t) &= P\mathbf{A}\bar{\mathbf{H}}^{\dagger}(t)\left(\mathbf{I}_{2N} + P\bar{\mathbf{H}}(t)\bar{\mathbf{H}}^{\dagger}(t)\right)^{-1} \\ &= P\mathbf{A}\left(\mathbf{I}_{2M} + P\bar{\mathbf{H}}^{\dagger}(t)\bar{\mathbf{H}}(t)\right)^{-1}\bar{\mathbf{H}}^{\dagger}(t) \end{aligned} \quad (4)$$

and $\mathbf{A} \in \mathbb{Z}^{2M\times 2M}$ is a full-rank integer matrix over $\mathbb{Z}_2$ chosen such that the effective noise in (3) is minimized. Here, an integer matrix $\mathbf{A}$ robust to the channel variation can be found by solving the following optimization problem [12], [18] with approximate search algorithms such as the Lenstra-Lenstra-Lovász (LLL) algorithm [35],

$$\underset{\mathbf{A}\in\mathbb{Z}^{2M\times 2M},\ \text{rank}(\mathbf{A})=2M}{\arg\min} \ \max_{m} ||\mathbf{D}^{-1/2}\mathbf{V}^{\dagger}\mathbf{a}_m||^2, \quad (5)$$

where $\mathbf{V}$ is an orthogonal matrix composed of the eigenvectors of

$$\mathbf{Q} = \left(\frac{1}{n}\sum_{t=1}^{n}\left(\mathbf{I}_{2M} + P\bar{\mathbf{H}}^{\dagger}(t)\bar{\mathbf{H}}(t)\right)^{-1}\right)^{-1} \quad (6)$$

as its columns, $\mathbf{D}$ is a diagonal matrix whose entries are the eigenvalues of $\mathbf{Q}$, and $\mathbf{a}_m \in \mathbb{R}^{2M\times 1}$ is the mth column vector in $\mathbf{A}^{\dagger}$.

To extract integer-linear combinations of codewords from $\tilde{\mathbf{y}}(t)$, the receiver performs a 'remapping' operation by scaling $\tilde{\mathbf{y}}(t)$ by $1/\alpha$ and then adding the offset β as in [36], which results in

$$\begin{aligned}\tilde{\tilde{\mathbf{y}}}(t) &= \frac{1}{\alpha}\tilde{\mathbf{y}}(t) + \beta\mathbf{A}\mathbf{1}_{2M\times 1} \\ &= \mathbf{A}\mathbf{x}_{\text{b}}(t) + \mathbf{z}_{\text{IF}}(t), \end{aligned} \quad (7)$$

where $\mathbf{z}_{\text{IF}}(t) := \frac{1}{\alpha}\left((\mathbf{F}_{\text{IF}}(t)\bar{\mathbf{H}}(t) - \mathbf{A})\bar{\mathbf{x}}(t) + \mathbf{F}_{\text{IF}}(t)\bar{\mathbf{z}}(t)\right)$ and

$$\mathbf{x}_{\text{b}}(t) := \left[\begin{array}{cccc} b_1(t) & b_2(t) & \cdots & b_{2M}(t) \end{array}\right]^{\dagger}.$$

Note that, after the remapping operation, a noisy version of the integer-linear sum of codewords can be observed at each SISO decoder i, i.e., $\tilde{\tilde{y}}_i(t) = \mathbf{a}_i^{\dagger}\mathbf{x}_{\text{b}}(t) + z_{\text{IF},i}(t)$ is observed at SISO decoder i, where

$$\left[\begin{array}{cccc} \mathbf{a}_i^{\dagger}\mathbf{x}_{\text{b}}(1) & \mathbf{a}_i^{\dagger}\mathbf{x}_{\text{b}}(2) & \cdots & \mathbf{a}_i^{\dagger}\mathbf{x}_{\text{b}}(n) \end{array}\right]$$

is an integer-linear sum of codewords, and $\tilde{\tilde{y}}_i(t)$ and $z_{\text{IF},i}(t)$ are the i-th elements in $\tilde{\tilde{\mathbf{y}}}(t)$ and $\mathbf{z}_{\text{IF}}(t)$, respectively. Since any integer-linear combination of codewords over $\mathbb{Z}_2^n$ is also itself a codeword when the same binary linear code is used across transmit antennas,

$$\left[\begin{array}{cccc} \mathbf{a}_i^{\dagger}\mathbf{x}_{\text{b}}(1) & \mathbf{a}_i^{\dagger}\mathbf{x}_{\text{b}}(2) & \cdots & \mathbf{a}_i^{\dagger}\mathbf{x}_{\text{b}}(n) \end{array}\right] \mod 2 \quad (8)$$

is also a codeword that can be directly decoded at decoder i. To this end, each SISO decoder i performs the modulo-2 operation on $\tilde{\tilde{y}}_i(t)$ and then attempts to recover the integer-linear combination (8) directly by calculating the log-likelihood-ratio (LLR) of $(\mathbf{a}_i^{\dagger}\mathbf{x}_b(t) \mod 2)$ for all $t \in [1:n]$. If the linear combinations of codewords at all the SISO decoders are successfully decoded, the original data streams can be recovered from the decoding outputs of the linear combinations by inverting $\mathbf{A}$ over $\mathbb{Z}_2$ in the absence of any noise components, as stated in [12].

C. Codeword-level MMSE-SIC

We now briefly review a well-known MMSE codeword-level SIC scheme [3]. As in the IF scheme, assume that each transmit antenna i sends an independent data stream $\mathbf{w}_i$. In contrast to the IF scheme, MMSE-SIC receivers sequentially decode for data streams by applying MMSE filters and then cancel out the decoded stream before decoding for the remaining streams. Specifically, the receiver first applies the MMSE filter given by

$$\begin{aligned}\mathbf{F}_{\text{MMSE}}(t) &= P\bar{\mathbf{H}}^{\dagger}(t)\left(\mathbf{I}_{2N} + P\bar{\mathbf{H}}(t)\bar{\mathbf{H}}^{\dagger}(t)\right)^{-1} \\ &= P\left(\mathbf{I}_{2M} + P\bar{\mathbf{H}}^{\dagger}(t)\bar{\mathbf{H}}(t)\right)^{-1}\bar{\mathbf{H}}^{\dagger}(t) \end{aligned} \quad (9)$$

which gives the effective SNR for stream $\mathbf{w}_k$ at resource element t as [12]

$$\begin{aligned}&\text{SNR}_{\text{MMSE},k}(t) \\ &= \frac{P\left(\mathbf{f}_{\text{MMSE},k}^{\dagger}(t)\bar{\mathbf{h}}_k(t)\right)^2}{||\mathbf{f}_{\text{MMSE},k}^{\dagger}(t)||^2 + P\sum_{i=1,i\neq k}^{M}\left(\mathbf{f}_{\text{MMSE},k}^{\dagger}(t)\bar{\mathbf{h}}_i(t)\right)^2}, \end{aligned} \quad (10)$$

where $\bar{\mathbf{h}}_i(t) \in \mathbb{R}^{2N\times 1}$ is the i-th column vector in $\bar{\mathbf{H}}(t)$ and $\mathbf{f}_{\text{MMSE},i}(t) \in \mathbb{R}^{2N\times 1}$ is the i-th column vector in $\mathbf{F}_{\text{MMSE}}^{\dagger}(t)$. For each sequential decoding, assume that the receiver attempts to recover the stream with the highest average effective SNR. Thus, the receiver decodes $\mathbf{w}_{i^*}$, where $i^* = \underset{i\in[1:2M]}{\arg\max}\ \frac{1}{n}\sum_{t=1}^{n}|\text{SNR}_{\text{MMSE},i}(t)|^2$, and then cancels its contribution from the received vector. The channel matrix then effectively becomes

$$\begin{aligned}&\bar{\mathbf{H}}_{i^*}(t) \\ &:= \left[\begin{array}{ccccccc} \bar{\mathbf{h}}_1(t) & \cdots & \bar{\mathbf{h}}_{i^*-1}(t) & \bar{\mathbf{h}}_{i^*+1}(t) & \cdots & \bar{\mathbf{h}}_{2M}(t) \end{array}\right]\end{aligned}$$

and the input vector becomes

$$\begin{aligned}&\bar{\mathbf{x}}_{i^*}(t) \\ &:= \left[\begin{array}{ccccccc} \bar{x}_1(t) & \cdots & \bar{x}_{i^*-1}(t) & \bar{x}_{i^*+1}(t) & \cdots & \bar{x}_{2M}(t) \end{array}\right]^{\dagger}\end{aligned}$$

for all $t \in [1:n]$. The receiver continues the sequential decoding process based on $\bar{\mathbf{H}}_{i^*}(t)$ and $\bar{\mathbf{x}}_{i^*}(t)$. This recursive process continues until all the streams are successfully decoded. If the decoding fails during the procedure, the receiver stops the decoding process and declares an error. Since there are $2M$ streams in total, the SIC operation across data streams can be performed up to $2M-1$ times.

Remark 1: Both IF and MMSE-SIC provide performance gains over MMSE by enhancing the effective SNR observed at each SISO decoder. Still, the approaches to achieve the gains are entirely different. The gain of IF mainly comes from the fact that the integer matrix can be freely chosen to minimize the effective noise, instead of merely using an identity matrix as in MMSE. In contrast to IF, even though the integer matrix is set as an identity matrix in MMSE-SIC, the effective SNR can be improved by SIC across data streams. Therefore, each of IF and MMSE-SIC is more suitable for a different channel environment, respectively. To be concrete, if the channel matrix is highly correlated (ill-conditioned), IF outperforms MMSE-SIC because noise amplification severely grows due to MMSE filtering in MMSE-SIC. MMSE-SIC, however, is more robust to the channel variation than IF. See the link-level simulation (LLS) results in [4, Section V] for more details. $\diamondsuit$

III. Successive IF and Successive Cancellation Integer Forcing with Practical Binary Codes

A. Successive IF with Binary Codes

Successive IF proposed in [15] can improve the basic IF [12] by combining IF sum decoding with SIC. Instead of decoding integer-linear combinations of codewords in parallel at all SISO decoders, the successive IF receiver sequentially recovers them one-by-one. After a linear combination of codewords is decoded, the receiver estimates the corresponding effective noise for the combination. It then performs SIC of it to reduce the effective noise for the remaining integer-linear combinations. The successive IF scheme can outperform the basic IF scheme due to this noise reduction. In this subsection, we explain how the ideas of successive IF can be implemented with practical binary codes instead of lattice codes. Furthermore, we describe the proposed modifications of successive IF to achieve better performance by combing successive IF with MMSE-SIC.

Recall the input-output relation after the remapping operation (7), which is given by

$$\begin{aligned}\tilde{\tilde{\mathbf{y}}}(t) &= \frac{1}{\alpha}\tilde{\mathbf{y}}(t) + \beta\mathbf{A}\mathbf{1}_{2M\times 1}\\ &= \mathbf{A}\mathbf{x}_{\mathrm{b}}(t) + \mathbf{z}_{\mathrm{IF}}(t).\end{aligned}$$

Based on $\tilde{\tilde{y}}_1(t) \mod 2$, $t \in [1:n]$, the proposed successive IF receiver first decodes

$$[\ \mathbf{a}_1^\dagger\mathbf{x}_{\mathrm{b}}(1) \quad \mathbf{a}_1^\dagger\mathbf{x}_{\mathrm{b}}(2) \quad \cdots \quad \mathbf{a}_1^\dagger\mathbf{x}_{\mathrm{b}}(n)\ ] \mod 2 \tag{11}$$

by SISO decoder 1. Next, assuming that (11) is successfully decoded, we design the receiver to estimate the effective noise $z_{\mathrm{IF},1}(t)$ when decoding (11), taking into account the modulo-2 operation, as follows:

$$\hat{z}_1(t) = \begin{cases} \hat{\hat{z}}_1(t) - 2 & \text{if } \hat{\hat{z}}_1(t) \geq 1,\\ \hat{\hat{z}}_1(t) & \text{otherwise,}\end{cases} \tag{12}$$

for all $t \in [1:n]$, where $\hat{z}_i(t)$ is the estimated version of $z_{\mathrm{IF},i}(t)$ and $\hat{\hat{z}}_1(t) := \tilde{\tilde{y}}_1(t) \mod 2 - (\mathbf{a}_1^\dagger\mathbf{x}_{\mathrm{b}}(t)) \mod 2$.

Then, as similar to [15], the receiver performs SIC of $\hat{z}_1(t)$ to have a less noisy channel for the remaining integer-linear combinations. Here, we assume that the condition $\hat{z}_i(t) = z_{\mathrm{IF},i}(t)$ is hold (see Remark 3 for further discussion). Note that the work of [15] was restricted to the block fading channel in which the channel is assumed to be fixed during the codeword length n. Therefore, we propose a modified SIC operation suitable for practical environments in which the channel can vary over the codeword length, that is, the receiver performs

$$\tilde{\tilde{\mathbf{y}}}^{(2)}(t) = \left(\tilde{\tilde{\mathbf{y}}}(t) \mod 2 - \left(\mathbf{l}_1(t)l_{1,1}^{-1}(t)\hat{z}_1(t)\right)\right) \mod 2 \tag{13}$$

for all $t \in [1:n]$, where the modulo-2 operation is owing to using binary codes and $\mathbf{l}_i(t)$ and $l_{i,j}(t)$ are the i-th column vector and the (i,j)-th element of $\mathbf{L}(t)$, respectively, where $\mathbf{L}(t) \in \mathbb{R}^{2M\times 2M}$ is a lower triangular matrix obtained from the following Cholesky decomposition

$$\mathbf{A}(\mathbf{I}_{2M} + P\bar{\mathbf{H}}^\dagger(t)\bar{\mathbf{H}}(t))^{-1}\mathbf{A}^\dagger = \mathbf{L}(t)\mathbf{L}^\dagger(t), \tag{14}$$

by extending [15, eqn. (16)] to the case in which the channel can change over the codeword length. From (14), we can see that $\mathbf{L}(t)$ is adapted to $\mathbf{H}(t)$ for each resource element t. Moreover, the effective noise variance for the second integer-linear combination is reduced to $Pl_{22}^2(t)$ due to the proposed SIC operation, which was $P(l_{22}^2(t) + l_{12}^2(t))$ for the basic IF scheme.[1] Therefore, the proposed successive IF scheme can improve the basic IF scheme.

Based on $\tilde{\tilde{\mathbf{y}}}^{(2)}(t)$, the proposed successive IF receiver now attempts to decode

$$[\ \mathbf{a}_2^\dagger\mathbf{x}_{\mathrm{b}}(1) \quad \mathbf{a}_2^\dagger\mathbf{x}_{\mathrm{b}}(2) \quad \cdots \quad \mathbf{a}_2^\dagger\mathbf{x}_{\mathrm{b}}(n)\ ] \mod 2$$

by following a similar step. This successive IF sum decoding procedure continues until all integer-linear combinations of codewords are decoded. If all the linear combinations are successfully decoded, the original data streams can be recovered from the decoding outputs of the linear combinations in the same way as the basic IF in Section II-B.

As in the basic IF, even when the channel varies, the integer matrix should also be fixed during the codeword length n for successive IF. Motivated by the approach in [15, Section IV], we propose an integer matrix $\mathbf{A}_{\mathrm{SIC}}$ for the successive IF receiver suitable for practical frequency-selective or time-varying channels, as stated in the following remark.

Remark 2: Following the same proof step in [15], it can be seen that a full-rank integer matrix $\mathbf{A}_{\mathrm{SIC}}$ over $\mathbb{Z}_2$ for the successive IF receiver for practical frequency-selective or time-varying channels can be obtained by solving the following optimization problem

$$\underset{\mathbf{A}_{\mathrm{SIC}}\in\mathbb{Z}^{2M\times 2M},\ \mathrm{rank}(\mathbf{A}_{\mathrm{SIC}})=2M}{\arg\min} \max_{k=1,2,\ldots,2M} \bar{l}_{k,k}^2 \tag{15}$$

where $\bar{l}_{i,j}$ is the (i,j)-th element of $\bar{\mathbf{L}} \in \mathbb{R}^{2M\times 2M}$, where $\bar{\mathbf{L}}$ is a lower triangular matrix obtained from the following Cholesky decomposition

$$\mathbf{A}_{\mathrm{SIC}}\mathbf{Q}^{-1}\mathbf{A}_{\mathrm{SIC}}^\dagger = \bar{\mathbf{L}}\bar{\mathbf{L}}^\dagger, \tag{16}$$

instead of solving (5). The above optimization problem (15) can be numerically solved by sequentially applying the LLL algorithm $2M$ times [37]. $\diamondsuit$

Remark 3: As discussed in [15, Section III-B], the successive IF scheme requires that the condition

$$\hat{z}_i(t) = z_{\mathrm{IF},i}(t) \tag{17}$$

is satisfied. In [15], a nested lattice code with Poltyrev good was assumed to be used to satisfy the condition (17). However, as will be shown below, it turns out that $\Pr(\hat{z}_i(t) \neq z_{\mathrm{IF},i}(t))$ is quite small under the proposed coding scheme, leading that the successive IF scheme can be implemented via off-the-shelf codes.

[1] To avoid duplication of explanation, we refer to [15, Section III] for detailed derivation.

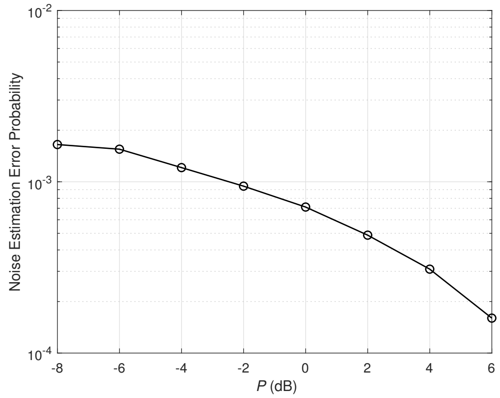


Fig. 1. Noise estimation error probability vs. P in the TDL-A channel with long RMS delay spread ($\delta = 300\ ns$) when $M = N = 4$.

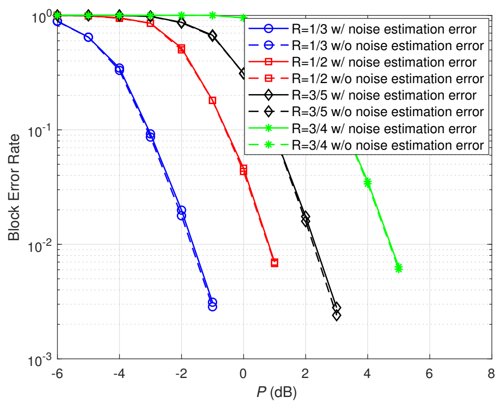


Fig. 2. BLER vs. P in the TDL-A channel with long RMS delay spread ($\delta = 300\ ns$) when $M = N = 4$ with and without the noise estimation error for different code rates.

As explained in Section III-A, in the proposed successive IF scheme, the receiver attempts to estimate $\hat{z}_i(t)$ after the i-th integer-linear combination of streams is successfully decoded. Observe that this noise estimation error occurs only if $|z_{\text{IF},i}(t)| \geq 1$, since $|\hat{z}_i(t)| \leq 1$ as defined in (12). Fig. 1 plots the probability of noise estimation error versus P in the tapped delayed line (TDL)-A channel model with long root mean square (RMS) delay spread (RMS delay spread $\delta = 300\ ns$) and the correlation factor[2] $\rho = 0$ defined in the third generation partnership project (3GPP) standard [39] when $M = N = 4$, demonstrating that the probability of estimation error is minimal. We also plot the block error rate (BLER) versus P in the same channel model with and without the noise estimation error for different code rates in Fig. 2.[3] The result demonstrates that the presence of noise estimation error has a negligible effect on the BLER indeed. We observe a similar trend in all the channels considered in this paper. [4]

[2]The channel correlation factor ρ represents the correlation between adjacent antennas [38].

[3]In the case of "without the noise estimation error," it is assumed that the noise estimation is perfectly carried out by a genie.

[4]As demonstrated in Fig. 5(b), although the probability of noise estimation error increases slightly when the RMS delay spread is extremely long, the consequent BLER performance reduction is not considerable.

Therefore, the successive IF scheme can be implemented via off-the-shelf binary codes instead of lattice codes with only minimal performance loss. $\Diamond$

B. *Successive Cancellation Integer Forcing (SC-IF)*

Although the successive IF scheme, which improves the basic IF scheme, is implementable via practical binary codes as shown in the previous subsection, it is relatively degraded compared to MMSE-SIC when the channel variation becomes severe as in the basic IF scheme, which will be shown in Section IV. This degradation is due to the inherent characteristic of IF sum decoding that an integer matrix needs to be fixed during the codeword length, which is also applied to the successive IF scheme, even when channels vary considerably over resource elements. Specifically, an integer matrix cannot be optimized for each channel realization. It should be fixed during the codeword length to ensure that integer-linear sums of codewords are themselves codewords. Therefore, as the channel variation increases, the difference between the integer matrix obtained by solving (15) and the integer matrix optimized for each channel realization, i.e., the integer matrix minimizing the effective noise for each channel realization, becomes inevitably severe. It results in degraded performance of successive IF as in the basic IF.

To overcome this challenge, we propose a novel enhanced scheme, namely, *successive cancellation integer forcing (SC-IF) scheme*, by combining the successive IF scheme with MMSE-SIC to derive each benefit. The key idea is that the receiver performs successive IF sum decoding only for chosen data streams instead of all the streams while performing individual MMSE-SIC decoding for the other remaining streams. Specifically, the receiver first sequentially and individually decodes for some data streams in conjunction with SIC. Then the remaining streams are recovered by successive IF sum decoding. The following two facts motivate our strategy of combining successive IF with MMSE-SIC: 1) MMSE-SIC is robust to the channel variation as discussed in Remark 1; and 2) MMSE-SIC across streams improves not only the effective SNRs of individual codewords but also those of codeword sums for IF decoding by allowing an integer matrix $\mathbf{A}$ to be updated appropriately based on a reduced channel matrix, as will be shown in Remark 4.

In the rest of this subsection, we will explain how to classify each stream into streams to which successive IF sum decoding and MMSE-SIC individual decoding are applied in detail.

Our proposed SC-IF scheme consists of the following steps:

1) Initially, set $\mathcal{W}_{\text{IF}} = [1:2M]$, where $\mathcal{W}_{\text{IF}}$ is the set of candidate streams that may be decoded by successive IF sum decoding.
2) The receiver calculates $\mathbf{L}(t)$ as in (14) for all $t \in [1:n]$ based on $(\bar{\mathbf{H}}(1), \bar{\mathbf{H}}(2), \cdots, \bar{\mathbf{H}}(n))$. We assume that the integer matrix is obtained from (15). Note that $1/l_{i,i}^2(t)$ denotes the effective SNR for successive IF sum decoding at SISO decoder i at resource element t. Then the receiver calculates the mod-2 capacity $C_{1/l_{i,i}^2(t)}$ based on $1/l_{i,i}^2(t)$ for all $t \in [1:n]$ and $i \in \mathcal{W}_{\text{IF}}$, which

is an achievable rate of this channel [4], where C_{σ^2} is calculated as

$$C_{\sigma^2} = 1 + h\left(\mathbb{Z}, \sigma^2\right) - h\left(2\mathbb{Z}, \sigma^2\right) \tag{18}$$

where

$$h\left(\eta\mathbb{Z}, \sigma^2\right) = -\int_{-\eta/2}^{\eta/2} f_{\eta\mathbb{Z},\sigma^2}(n') \log_2\left(f_{\eta\mathbb{Z},\sigma^2}(n')\right) dn',$$

$$f_{\eta\mathbb{Z},\sigma^2}(n') := \sum_{b\in\eta\mathbb{Z}} g_{\sigma^2}(n'+b),\ n' \in [-\eta/2, \eta/2],\ \eta \in \mathbb{Z},$$

and $g_{\sigma^2}(\theta)$ is the Gaussian probability density function (pdf) with zero mean and noise variance σ^2, i.e., $g_{\sigma^2}(\theta) = \left(2\pi\sigma^2\right)^{-1/2} e^{-\theta^2/2\sigma^2}$. We refer to [4], [40] for detailed derivations to avoid duplication. After that, the receiver obtains the average mod-2 capacity over resource elements, i.e., $\bar{C}_{\text{IF},i} = \frac{1}{n}\sum_{t=1}^{n} C_{1/l_{i,i}^2(t)}$ for all $i \in \mathcal{W}_{\text{IF}}$. Then the receiver calculates

$$m_{\text{IF},i} = C_{\text{BPSK}}^{-1}\left(\bar{C}_{\text{IF},i}\right) - C_{\text{BPSK}}^{-1}(r), \tag{19}$$

where $C_{\text{BPSK}}(x)$ denotes the capacity for binary phase shift keying (BPSK) modulation over the additive white Gaussian noise (AWGN) channel with SNR x, which is given by [41], [42]

$$C_{\text{BPSK}}(x) = 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi/x}} e^{\frac{-(t-1)^2 x}{2}} \log_2\left(1 + e^{-2tx}\right) dt, \tag{20}$$

and $C_{\text{BPSK}}^{-1}(y) = x$ denotes the inverse function of $C_{\text{BPSK}}(x) = y$. The metric (19) is proposed to approximate the average SNR margin obtained with successive IF compared to the allocated code rate r. The motivation to consider the capacity of BPSK modulation is that this value can serve as an asymptotic performance upper bound for binary codes, and 5G LDPC codes to be considered in Section V provide good performance close to the bound.

3) The receiver calculates

$$\dot{\mathbf{Q}}(t) = \left(\mathbf{I}_{|\mathcal{W}_{\text{IF}}|} + P\bar{\mathbf{H}}^{\dagger}(t)\bar{\mathbf{H}}(t)\right)^{-1} \tag{21}$$

for all $t \in [1:n]$ based on $(\bar{\mathbf{H}}(1), \bar{\mathbf{H}}(2), \cdots, \bar{\mathbf{H}}(n))$. Let $q_{i,j}(t)$ denote the (i,j)-th element of $\dot{\mathbf{Q}}(t)$. Note that $1/q_{i,i}^2(t)$ denotes the effective SNR for MMSE individual decoding at SISO decoder i at resource element t. Then, as similar to Step 2), the receiver calculates the mod-2 capacity $C_{1/q_{i,i}^2(t)}$ for all $t \in [1:n]$ and $i \in \mathcal{W}_{\text{IF}}$ and then obtains the average mod-2 capacity over resource elements $\bar{C}_{\text{MMSE},i} = \frac{1}{n}\sum_{t=1}^{n} C_{1/q_{i,i}^2(t)}$ for all $i \in \mathcal{W}_{\text{IF}}$. Then the receiver calculates

$$m_{\text{MMSE},i} = C_{\text{BPSK}}^{-1}\left(\bar{C}_{\text{MMSE},i}\right) - C_{\text{BPSK}}^{-1}(r) \tag{22}$$

in order to approximate the average SNR margin obtained with MMSE compared to the allocated code rate r.

Algorithm 1 Proposed SC-IF Scheme

Initialization: Set $\mathcal{W}_{\text{IF}} = [1:2M]$
while $|\mathcal{W}_{\text{IF}}| \geq 1$ **do**
  Update $\mathbf{A}_{\text{SIC}}$, $\mathbf{L}(t)$, and $\dot{\mathbf{Q}}(t)$ based on the current $\bar{\mathbf{H}}(t)$
  for $i \in \mathcal{W}_{\text{IF}}$ **do**
    Calculate $\bar{C}_{\text{IF},i}$ and $\bar{C}_{\text{MMSE},i}$
    Calculate $m_{\text{IF},i}$ and $m_{\text{MMSE},i}$
  end for
  $i^* \leftarrow \arg\min_{i\in\mathcal{W}_{\text{IF}}} m_{\text{IF},i}$
  $j^* \leftarrow \arg\max_{j\in\mathcal{W}_{\text{IF}}} m_{\text{MMSE},j}$
  if $m_{\text{MMSE},j^*} > m_{\text{IF},i^*}$ **then**
    Receiver recovers $\mathbf{w}_{j^*}$ and cancels out its contribution from the received vector
    $\mathcal{W}_{\text{IF}} \leftarrow \mathcal{W}_{\text{IF}} \backslash \{j^*\}$, $\bar{\mathbf{H}}(t) \leftarrow \bar{\mathbf{H}}_{j^*}(t)$
  else
    Follow the successive IF decoding procedure explained in Section III-A for all the remaining streams
    End the entire process
  end if
end while

4) First, the receiver finds the worst channel for successive IF based on the metric (19), that is, the receiver finds i^* such that $i^* = \arg\min_{i\in\mathcal{W}_{\text{IF}}} m_{\text{IF},i}$. Second, it finds the best channel for MMSE based on the metric (22), that is, it finds j^* such that $j^* = \arg\max_{j\in\mathcal{W}_{\text{IF}}} m_{\text{MMSE},j}$. Note that the worst channel is the bottleneck for the overall MIMO transmission in the case of successive IF sum decoding while the receiver attempts to recover the stream with the best channel in the case of MMSE-SIC individual decoding. Hence, the receiver compares the SNR margin of the worst channel of IF with that of the best channel of MMSE. Then the receiver performs one of the following procedures:
   a) If
   $$m_{\text{MMSE},j^*} > m_{\text{IF},i^*}, \tag{23}$$
   then the receiver performs MMSE-SIC individual decoding, i.e., the receiver decodes for stream $\mathbf{w}_{j^*}$ and cancels out its contribution from the received vector. Next, if $|\mathcal{W}_{\text{IF}}| \neq 1$, the receiver modifies the set $\mathcal{W}_{\text{IF}}$ by removing j^* from it, that is, $\mathcal{W}_{\text{IF}} \leftarrow \mathcal{W}_{\text{IF}} \backslash \{j^*\}$, and repeats Steps 2) to 4) by replacing the channel matrix $\bar{\mathbf{H}}(t)$ with $\bar{\mathbf{H}}_{j^*}(t)$ and the input vector $\bar{\mathbf{x}}(t)$ with $\bar{\mathbf{x}}_{j^*}(t)$ for all $t \in [1:n]$. On the other hand, if $|\mathcal{W}_{\text{IF}}| = 1$, the receiver recovers one remaining stream and finishes the entire process.
   b) Otherwise, the receiver follows the successive IF decoding procedure explained in the previous subsection for all the remaining streams and finishes the entire process.

The proposed SC-IF procedure is summarized in Algorithm 1.

Remark 4 (SNR improvement due to MMSE-SIC): Consider the case in which the condition (23) is satisfied. Without loss of generality, assume that $\mathbf{w}_1$ is individually decoded and successively canceled out by MMSE-SIC while the other streams are not yet decoded. Then, after MMSE-SIC, the effective channel matrix becomes $\bar{\mathbf{H}}_1(t)$ for all $t \in [1:n]$. Let $\mathbf{A} \in \mathbb{Z}^{2M\times 2M}$ and $\bar{\mathbf{A}} \in \mathbb{Z}^{(2M-1)\times(2M-1)}$ be the integer matrices obtained based on $(\bar{\mathbf{H}}(1), \bar{\mathbf{H}}(2), \cdots, \bar{\mathbf{H}}(n))$ and $(\bar{\mathbf{H}}_1(1), \bar{\mathbf{H}}_1(2), \cdots, \bar{\mathbf{H}}_1(n))$, respectively. In addition, denote the effective noise variances at SISO decoder $m \in [2:2M]$ based on

$$\big(\mathbf{A} \text{ and } (\bar{\mathbf{H}}(1), \cdots, \bar{\mathbf{H}}(n))\big)$$

and

$$\big(\bar{\mathbf{A}} \text{ and } (\bar{\mathbf{H}}_1(1), \bar{\mathbf{H}}_1(2), \cdots, \bar{\mathbf{H}}_1(n))\big)$$

by $\sigma_m^2(t)$ and $\bar{\sigma}_m^2(t)$, respectively. The effective noise variance for stream m is given by [12]

$$\begin{aligned}
\sigma_m^2(t) &= \frac{1}{n}\sum_{t=1}^{n} P\mathbf{a}_m^{\dagger}\big(\mathbf{I}_{2M} + P\bar{\mathbf{H}}^{\dagger}(t)\bar{\mathbf{H}}(t)\big)^{-1}\mathbf{a}_m \\
&\overset{(a)}{\geq} \frac{1}{n}\sum_{t=1}^{n} P\tilde{\mathbf{a}}_m^{\dagger}\Big(\mathbf{I}_{2M} + P\tilde{\mathbf{H}}^{\dagger}(t)\tilde{\mathbf{H}}(t)\Big)^{-1}\tilde{\mathbf{a}}_m \\
&= \frac{1}{n}\sum_{t=1}^{n} P\tilde{\tilde{\mathbf{a}}}_m^{\dagger}\Big(\mathbf{I}_{2M-1} + P\bar{\mathbf{H}}_1^{\dagger}(t)\bar{\mathbf{H}}_1(t)\Big)^{-1}\tilde{\tilde{\mathbf{a}}}_m \\
&\overset{(b)}{\geq} \frac{1}{n}\sum_{t=1}^{n} P\bar{\mathbf{a}}_m^{\dagger}\Big(\mathbf{I}_{2M-1} + P\bar{\mathbf{H}}_1^{\dagger}(t)\bar{\mathbf{H}}_1(t)\Big)^{-1}\bar{\mathbf{a}}_m \\
&= \bar{\sigma}_m^2(t), \quad \forall t \in [1:n],
\end{aligned}$$

where $\tilde{\mathbf{a}}_m = [\ 0 \quad \tilde{\tilde{\mathbf{a}}}_m^{\dagger}\ ]^{\dagger}$, $\tilde{\tilde{\mathbf{a}}}_m = [\ a_{m,2} \quad \cdots \quad a_{m,2M}\ ]^{\dagger} \in \mathbb{Z}^{(2M-1)\times 1}$ where $a_{i,j}$ is the (i,j)-th element in $\mathbf{A}$, $\bar{\mathbf{a}}_m$ is the mth column vector in $\bar{\mathbf{A}}^{\dagger}$, and $\tilde{\mathbf{H}}(t) = [\ \mathbf{0}_{2N\times 1} \quad \bar{\mathbf{H}}_1(t)\ ]$. Here, (a) can be easily verified from the matrix inversion lemma [43] and (b) is due to the fact that $\bar{\mathbf{a}}_m$ can be further optimized based on $(\bar{\mathbf{H}}_1(1), \bar{\mathbf{H}}_1(2), \cdots, \bar{\mathbf{H}}_1(n))$ while $\mathbf{a}_m$ is optimized based on $(\bar{\mathbf{H}}(1), \bar{\mathbf{H}}(2), \cdots, \bar{\mathbf{H}}(n))$. Therefore, we have shown that the MMSE-SIC operation in the proposed SC-IF can improve the average effective SNRs of the remaining streams compared to the plain successive IF scheme. $\Diamond$

Remark 5: Clearly, the proposed SC-IF scheme can recover the successive IF scheme in Section III-A and MMSE-SIC scheme in Section II-C by restricting $|\mathcal{W}_{\text{IF}}| = 2M$ and the integer matrix as an identity matrix for each sequential decoding, respectively. Moreover, the proposed SC-IF scheme can provide strictly better performance compared to the successive IF and MMSE-SIC schemes since MMSE-SIC across data streams can improve the average effective SNRs of remaining codewords as shown in Remark 4 and successive IF sum decoding can outperform MMSE-SIC individual decoding for many cases (unless the channel variation is higher than a certain threshold). Recall that, when the channel varies severely, the integer matrix obtained through (15) cannot guarantee optimality, unlike in a time-invariant environment. The proposed SC-IF is a practical approach that partially compensates for this problem by creating the opportunity to recalculate an integer matrix suitable for successive IF based on the reduced channel matrix after removing some streams with MMSE-SIC. On the other hand, when the channel variation is sufficiently small, the proposed SC-IF operates as successive IF since the integer matrix obtained through (15) provides good performance, and the condition (23) is rarely satisfied. $\Diamond$

Remark 6 (Receiver complexity comparison): The symbol detection complexity of MMSE and MMSE-SIC is given by $O(NM^2 + M^3)$ because the computational complexity of the MMSE filter operation (9) is $O(NM^2 + M^3)$ [44], [45]. For IF, the receiver needs to find an appropriate integer matrix by using the LLL algorithm in addition to the MMSE filter operation to calculate (4), which requires the computational complexity of $O(NM^3 \log M)$ [46], but this complexity increment is negligible since the LLL algorithm is required to be performed only once during the codeword length n, where $n \gg M$ or N. Therefore, the symbol detection complexity of IF is given by $O(NM^2 + M^3)$ [4], the same as the MMSE and MMSE-SIC cases. In the same manner, in the case of successive IF, although the computational complexity of finding a proper integer matrix increases as $O(NM^4 \log M)$ since the LLL algorithm needs to be performed $2M$ times, the symbol detection complexity of successive IF is also given by $O(NM^2 + M^3)$. For SC-IF, the worst computational complexity slightly increases as $O(NM^3 + M^4)$ since $\mathbf{A}_{\text{SIC}}$, $\mathbf{L}(t)$, and $\mathbf{Q}_1(t)$ need to be calculated at most $2M$ times.[5] However, it is much smaller than the computational complexity of ML detection given by $O\left(M|\mathcal{X}_L|^M\right)$ [44], [45], which becomes practically infeasible, especially when L or M is large. $\Diamond$

So far, we have assumed 4-QAM transmission, i.e., $L = 2$. To extend the proposed successive IF and SC-IF schemes to higher-order modulation ($L > 2$) with off-the-shelf binary linear codes, we adopt the MLC strategy as in [4]. We will briefly explain how to employ MLC for higher-order modulation in the next section and evaluate the performance of the proposed successive IF and SC-IF schemes in various aspects in Section V.

IV. Extension to Higher Modulation

We briefly explain MLC for successive IF and SC-IF, in which multilevel encoding (MLE) in conjunction with the natural mapping is employed on the transmitter side and multistage decoding (MSD) in conjunction with IF sum decoding is employed on the receiver side. MLC with the natural mapping enables the use of binary codes for IF sum decoding when $L > 2$ as explained in [4] because the modulo-2 operation at the receiver can separate received codewords into multiple independently encoded binary linear codewords according to levels regardless of the modulation order L. Therefore, each SISO decoder can reliably recover an integer-linear combination of binary codewords in each level up to the code's noise tolerance. To avoid the duplication of explanation, we will focus on the modifications introduced by MLC.

1) Transmitter Side: Consider the case where $L \geq 2$ for the MLE strategy [47]–[50]. The stream $\mathbf{w}_i$ of transmit antenna i is now divided into $L/2$ sub-data streams $\mathbf{w}_{j,i} \in \mathbb{Z}_2^{\kappa_j \times 1}$ with

[5] However, when the performance of successive IF is not significantly lower than that of MMSE-SIC, the number of iterations Algorithm 1 performs can be much less than $2M$.

length κ_j for $j \in [1 : L/2]$. Then $\mathbf{w}_{j,i}$ is mapped to the length-n binary linear codeword $\mathbf{b}_{j,i} \in \mathbb{Z}_2^{n\times 1}$ with the code rate $r_j = \frac{\kappa_j}{n}$ by the channel encoder at transmit antenna i, where j denotes the level of a transmitted bit, indicating the bit position conveyed in a modulated symbol. Assume that each encoder employs the same linear code for the same level, which is required for IF sum decoding. The code rate for each level is chosen according to the *capacity design rule* similar to [47]–[50]. The total normalized rate is given by $r_{\text{total}} = \frac{2}{L}\sum_{j=1}^{L/2} r_j$.

As in [4], the natural labeling is used for mapping between encoded bits and signal constellation points. The constellation points in the natural mapping are ordered according to the symbol indices, i.e.,

$$\bar{x}_i(t) = \alpha\left(\sum_{j=1}^{L/2} 2^{j-1} b_{j,i}(t) - \beta\right),$$

where $\bar{x}_i(t) \in \text{Re}(\mathcal{X}_L)$ is the modulation symbol of transmit antenna i sent at resource element t, $\alpha = 2\sqrt{P}((2^L - 1)/3)^{-1/2}$, and $\beta = (2^{L/2} - 1)/2$.

2) Receiver Side: We first consider the basic IF and successive IF receivers with MLC. As in [4], the basic IF receiver and successive IF receiver employ sequential decoding across levels, namely, MSD [47]–[50], in conjunction with IF sum decoding as follows. As explained in [4], by applying an IF filter and the remapping operation, the effective channel output vector (7) now becomes

$$\begin{aligned}\tilde{\tilde{\mathbf{y}}}_{\text{level},1}(t) &= \frac{1}{\alpha}\tilde{\mathbf{y}}(t) + \beta\mathbf{A}\mathbf{1}_{2M\times 1} \\ &= \mathbf{A}\sum_{j=1}^{L/2} 2^{j-1}\mathbf{x}_{\text{b},j}(t) + \mathbf{z}_{\text{IF},1}(t),\end{aligned} \tag{24}$$

where $\mathbf{z}_{\text{IF},1}(t) := \frac{1}{\alpha}\left((\mathbf{F}_{\text{IF}}(t)\bar{\mathbf{H}}(t) - \mathbf{A})\bar{\mathbf{x}}(t) + \mathbf{F}_{\text{IF}}(t)\bar{\mathbf{z}}(t)\right)$ and

$$\mathbf{x}_{\text{b},j}(t) := \begin{bmatrix} b_{j,1}(t) & b_{j,2}(t) & \cdots & b_{j,2M}(t) \end{bmatrix}^\dagger$$

consisting of encoded bits for level j. The modulo-2 operation can separately recover codewords in each level because of the natural mapping. Applying the modulo-2 operation on the effective channel output vector $\tilde{\tilde{\mathbf{y}}}_{\text{level},1}(t)$ in (24), all the encoded codewords except level-1 codewords are completely removed. Hence, each SISO decoder sees a noisy version of the integer-linear combination of level-1 codewords over $\mathbb{Z}_2^n$, which is also a codeword itself and therefore can be directly decoded up to the code's noise tolerance. In this way, the receiver can perform IF decoding and also successive IF decoding for level-1 codewords as explained in Sections II-B and III-A, respectively. If the decoding operation for level-1 codewords is successful, the receiver cancels out their contributions from $\tilde{\tilde{\mathbf{y}}}_{\text{level},1}(t)$, $\forall t \in [1 : n]$, and scales by $1/2$ to get

$$\tilde{\tilde{\mathbf{y}}}_{\text{level},2}(t) = \mathbf{A}\sum_{j=2}^{L/2} 2^{j-2}\mathbf{x}_{\text{b},j}(t) + \mathbf{z}_{\text{IF},2}(t), \tag{25}$$

where

$$\begin{aligned}\mathbf{z}_{\text{IF},2}(t) = \frac{1}{2\alpha}\Bigg((\mathbf{F}_{\text{IF}}(t)\bar{\mathbf{H}}(t) - \mathbf{A})\alpha\left(\sum_{j=2}^{L/2} 2^{j-1}\mathbf{x}_{\text{b},j}(t) - \beta\right) \\ + \mathbf{F}_{\text{IF}}(t)\bar{\mathbf{z}}(t)\Bigg).\end{aligned} \tag{26}$$

The receiver then can attempt to decode for level-2 codewords by following the same steps as in the decoding operation of level-1 codewords. The recursive procedure continues until data streams up to level $L/2$ are decoded. For more details, see [4].

Second, let us consider the MMSE-SIC receiver with MLC. Recall that the MMSE-SIC receiver sequentially decodes for each data stream. In the case of the proposed MMSE-SIC with MLC, SIC is first performed across levels for a given data stream and then across data streams. Specifically, when decoding for stream $\mathbf{w}_i$ in MLC, the MMSE-SIC receiver attempts to recover all the sub-streams of $\mathbf{w}_i$ from levels 1 to $L/2$, that is, it sequentially decodes for $\mathbf{w}_{j,i}$ for all $j \in [1 : L/2]$ via MSD before decoding for another stream $\mathbf{w}_k$ not yet decoded for $k \neq i$.

Finally, let us see how to modify the proposed SC-IF scheme when extending to MLC. By extending Steps 2) and 3) of the proposed SC-IF scheme described in Section III-B to the multi-level case, the receiver now calculates the average mod-2 capacity for each level $j \in [1 : L/2]$ for each iteration. Note that as explained above, the received codewords can be easily separated according to levels by the modulo-2 operation and MSD at the receiver since MLE with the natural mapping is used at the transmitter. Hence, the average mod-2 capacity for each level can be obtained in a way similar to the level-1 case. To be specific, for a given iteration, the receiver calculates the average mod-2 capacities of IF and MMSE at level j seen at SISO decoder i, denoted by $\bar{C}_{\text{IF},j,i}$ and $\bar{C}_{\text{MMSE},j,i}$, respectively, for all i and j. In particular, $\mathbf{A}_{\text{SIC}}$, $\mathbf{L}(t)$, and $\mathbf{Q}(t)$ are updated for each level j after performing MSD for the previous levels $1, 2, \ldots, j-1$, and $\bar{C}_{\text{IF},j,i}$ and $\bar{C}_{\text{MMSE},j,i}$ can be obtained by substituting the resulting effective SNRs into (18). Next, the receiver calculates

$$m_{\text{IF}} = \min_i \min_j \left(C_{\text{BPSK}}^{-1}(\bar{C}_{\text{IF},j,i}) - C_{\text{BPSK}}^{-1}(r_j)\right), \tag{27}$$

$$m_{\text{MMSE}} = \max_i \min_j \left(C_{\text{BPSK}}^{-1}(\bar{C}_{\text{MMSE},j,i}) - C_{\text{BPSK}}^{-1}(r_j)\right). \tag{28}$$

Note that in (27) and (28), for given i, we consider the minimum SNR margin with respect to levels, motivated by the fact that the receiver wishes to decode all levels of streams correctly. Then, in Step 4), if $m_{\text{MMSE}} > m_{\text{IF}}$, the receiver performs MMSE-SIC individual decoding with MLC, i.e., the receiver decodes for all the sub-streams of $\mathbf{w}_{i^*}$ from levels 1 to $L/2$ and cancels out their contributions from the received vector, where

$$i^* = \operatorname*{argmax}_i \min_j \left(C_{\text{BPSK}}^{-1}(\bar{C}_{\text{MMSE},j,i}) - C_{\text{BPSK}}^{-1}(r_j)\right).$$

Otherwise, the receiver performs the successive IF decoding procedure with the aforementioned MSD for all the remaining streams and finishes the entire process. The other parts follow

TABLE I
RATE ALLOCATION OF MULTILEVEL CODING FOR EACH LEVEL

Modulation order	Rate allocation
4-QAM ($L = 2$)	0.6
16-QAM ($L = 4$)	0.36, 0.84
64-QAM ($L = 6$)	0.22, 0.68, 0.9
256-QAM ($L = 8$)	0.05, 0.6, 0.85, 0.9

TABLE II
EVALUATION ASSUMPTIONS FOR LLS

Parameter	Assumption
Waveform	Orthogonal frequency division multiplexing (OFDM)
Carrier frequency	4 GHz
System bandwidth	10 MHz
Number of allocated resource blocks (RBs)	20
Channel model	TDL-A [39] (mobility: 3km/h)
Channel estimation	Perfect at the receiver, unknown to the transmitter
Channel code	5G NR LDPC, $r_{\text{total}} = 0.6$
Code length n	2640
MIMO configuration	4×4 MIMO, i.e., $M = N = 4$

the same approach as in the level-1 case described in Section III-B.

Remark 7: Note that for higher modulation, i.e., $L > 2$, the SIC operations of MMSE-SIC and successive IF in the proposed SC-IF are different. In the case of successive IF, SIC is first performed across data streams for given level and then across levels. On the other hand, in the case of MMSE-SIC, SIC is first performed across levels for a given data stream and then across data streams. $\Diamond$

Remark 8: Our proposed SC-IF system is easily adaptable to certain multi-user scenarios. Consider a multi-user uplink channel with several transmitters (users) and a single receiver. By individually encoding all users' data streams with the same linear code, the proposed SC-IF approach can be used to recover streams of multiple users at the receiver. Additionally, when combined with a rate splitting method [51] that divides each data stream into common and private sub-streams, where the private stream is recovered by the dedicated receiver only and the common stream is required to be recovered by all receivers, the proposed SC-IF scheme can be used to manage inter-user interference in multi-user downlink or interference channels. For additional information, see [28]–[30]. $\Diamond$

V. SIMULATION RESULTS

We now evaluate the LLS performance of the proposed successive IF and SC-IF schemes in various aspects, including higher-order modulation transmission ($L > 2$). Here, we mainly focus on the cases in which the channel variation is severe in the frequency domain (e.g., TDL-A channel with long RMS delay spread $\delta = 300$ ns) to emphasize the robustness of the proposed SC-IF to the channel variation.[6] Assume that 5G LDPC codes for eMBB with $r_{\text{total}} = 0.6$ are employed for all simulations,[7] where the rate allocation for each level is given in Table I. The other assumptions for LLS are stated in Table II. In addition, the BLER performance of ML detection is also plotted for the 4-QAM and 16-QAM cases for comparison.[8]

The results of all the simulations in Sections V-A, V-B, and V-C demonstrate that the proposed SC-IF scheme strictly outperforms both successive IF and MMSE-SIC over most of the entire SNR range regardless of the values of a channel variation and a channel correlation parameter.

A. Performance Analysis with respect to the Modulation Order

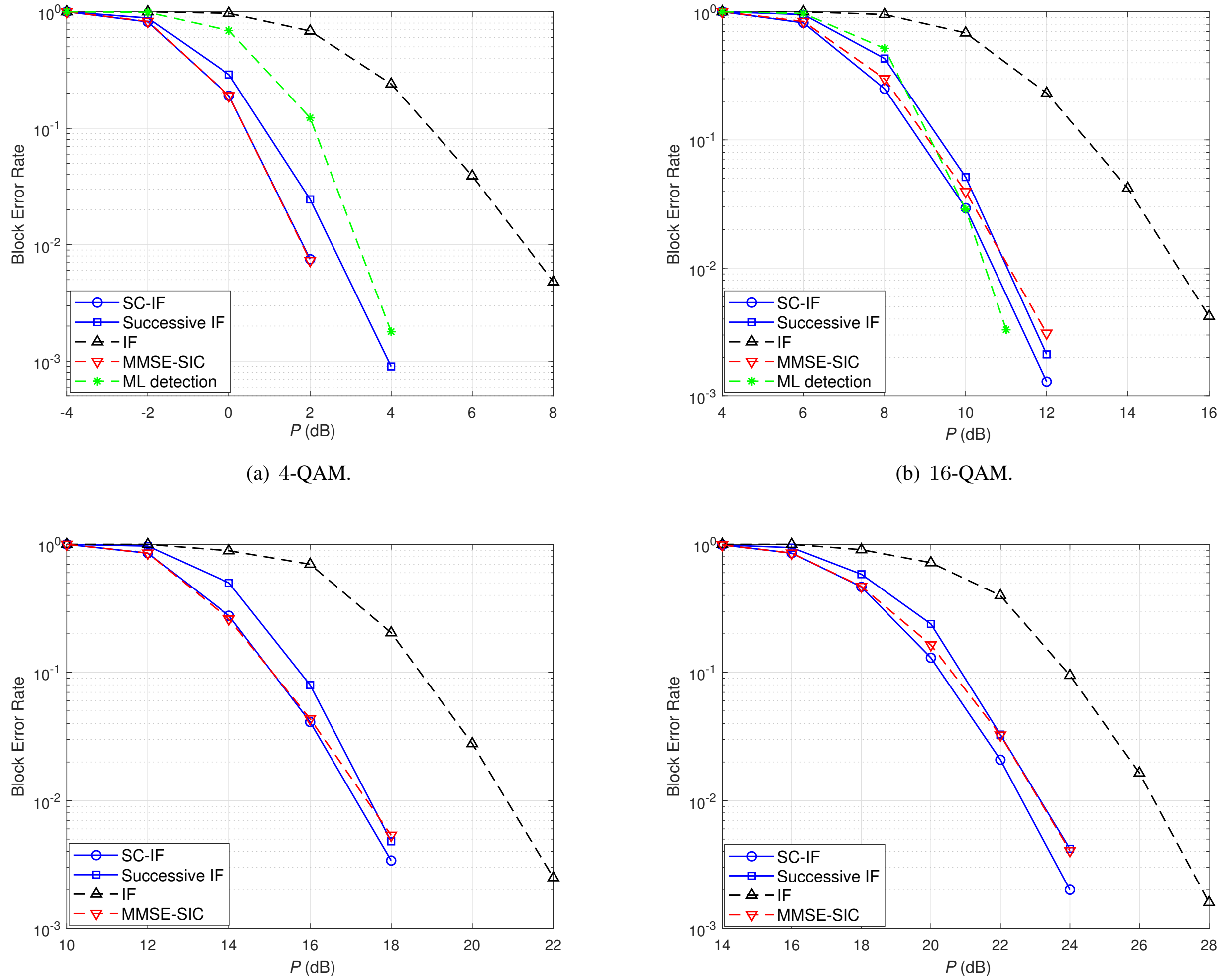
We evaluate the BLER performance of the proposed SC-IF, successive IF, IF, and MMSE-SIC schemes with respect to the modulation order. Here, we assume the TDL-A channel with long RMS delay spread ($\delta = 300$ ns) at $\rho = 0$. As shown in Fig. 3, simulation results for the BLERs of 4-QAM, 16-QAM, 64-QAM, and 256-QAM transmission of all the MIMO schemes demonstrate that MMSE-SIC outperforms IF and successive IF due to the severe channel variation. However, the proposed SC-IF scheme can outperform all the other linear receiver schemes over the entire SNR range for all modulation orders. Furthermore, except for the 4-QAM case, the proposed SC-IF scheme strictly outperforms both successive IF and MMSE-SIC. For example, SC-IF exceeds successive IF and MMSE-SIC by 0.5 dB at BLER $= 0.01$ in the case of 256-QAM.

In addition, it is shown that SC-IF and successive IF outperform the ML detection in the case of 4-QAM, while the trend is reversed at BLER $= 0.01$ in the case of 16-QAM. However, for the 16-QAM case, the performance gap is small enough from a practical point of view given the fact

[6]When the channel variation is not that severe, the performance of basic IF and successive IF becomes much better than that of MMSE-SIC in general (for example, refer to Fig. 5(a) for performance comparison in the TDL-A channel with very short RMS delay spread $\delta = 10$ ns). For more details about the performance evaluation of basic IF when the channel variation is relatively low, see [4, Section V].

[7]Note that our scheme can be implemented with any binary linear codes.

[8]The 64-QAM and 256-QAM cases are excluded due to their high computational complexity.



(a) 4-QAM. (b) 16-QAM.

(c) 64-QAM. (d) 256-QAM.

Fig. 3. BLER performance comparison for 4-QAM, 16-QAM, 64-QAM, and 256-QAM in the TDL-A channel with long RMS delay spread ($\delta = 300$ ns) at $\rho = 0$.

that the computational complexity of SC-IF or successive IF is much smaller than that of the ML detection.

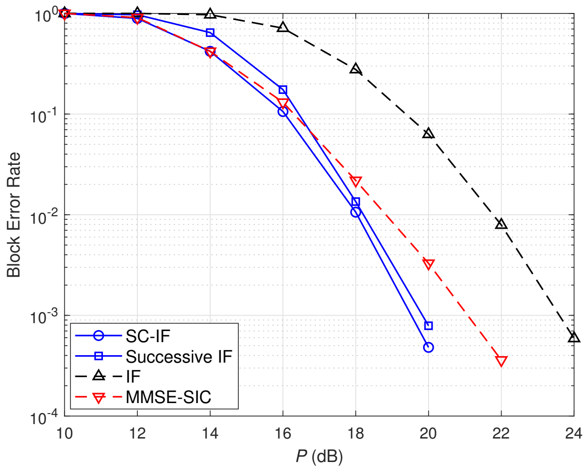
Remark 9: Notably, the ML detection receiver's performance gives an upper bound for the entire SNR range when no codeword-level SIC operation is done; for example, its performance can serve as an upper bound for the plain IF receiver's performance. However, because codeword-level SIC operations can raise the effective SNRs of the remaining streams, the SC-IF, successive IF, and MMSE-SIC can outperform the ML detection receiver in some instances, as the ML detection receiver is unable to achieve such gain. $\Diamond$

B. Performance Analysis with respect to the Channel Correlation

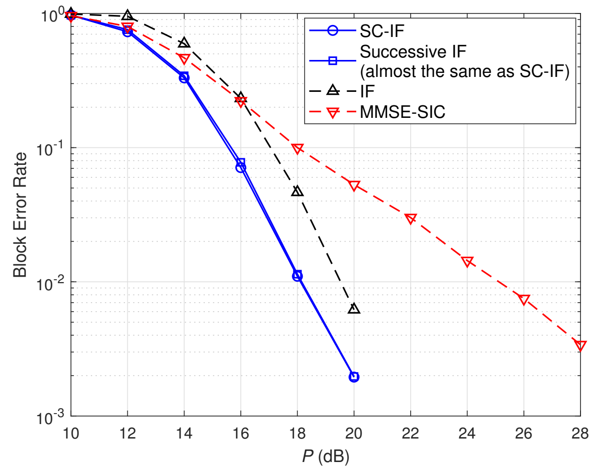
Now we evaluate the BLER performance with respect to the channel correlation. Simulation results for the BLERs of the proposed SC-IF, successive IF, IF, and MMSE-SIC schemes of 64-QAM transmission in the TDL-A channel with long RMS delay spread at $\rho = 0.3$ and $\rho = 0.6$ are shown in Fig. 4. By comparing Figs. 3(c), 4(a), and 4(b), it is demonstrated that the considered IF schemes eventually outperform MMSE-SIC as the channel correlation increases. This is because, as the channel becomes more correlated, the channel matrix becomes nearly singular; thus, significant noise amplification occurs during the channel inversion for decoupling in the case of MMSE-SIC. If the channel correlation is greater than a certain threshold, the proposed SC-IF scheme provides almost the same performance as successive IF (see Fig. 4(b)). A similar trend can also be found under other channel correlation models, such as the Rician channel model [3].

C. Performance Analysis with respect to the Channel Variation

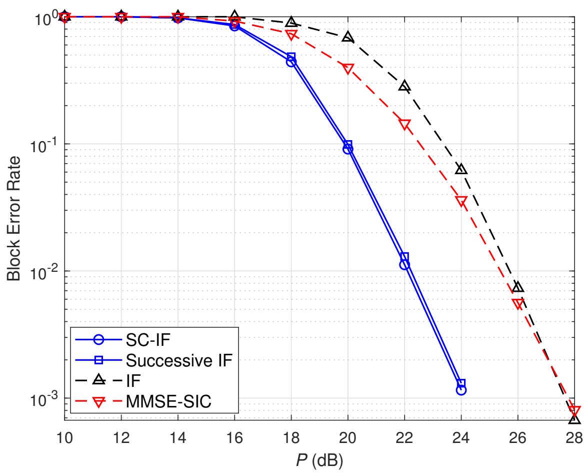
Finally, we evaluate the BLER performance with respect to the channel variation. Simulation results for the BLERs of the proposed SC-IF, successive IF, IF, and MMSE-SIC schemes of 64-QAM transmission in the TDL-A channels with very short RMS delay spread ($\delta = 10$ ns) and very long RMS delay spread ($\delta = 1000$ ns) at $\rho = 0$ are shown in Fig. 5. By comparing Figs. 5(a), 3(c), and 5(b), it is demonstrated that,



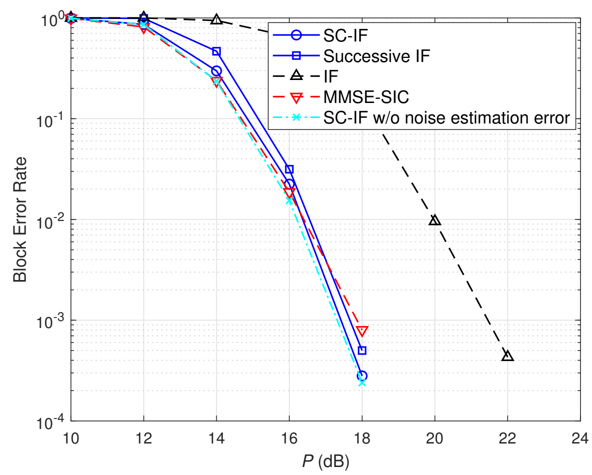
(a) $\rho = 0.3$.



(a) TDL-A channel with very short RMS delay spread ($\delta = 10\ ns$).



(b) $\rho = 0.6$.



(b) TDL-A channel with very long RMS delay spread ($\delta = 1000\ ns$).

Fig. 4. BLER performance comparison for 64-QAM in the TDL-A channel with long RMS delay spread ($\delta = 300\ ns$).

Fig. 5. BLER performance comparison for 64-QAM in the TDL-A channels with very short spread and very long RMS delay spread when $\rho = 0$.

when the channel variation is severe, the performance loss of IF and successive IF increases so that MMSE-SIC eventually outperforms them. On the other hand, the proposed SC-IF scheme can provide almost the same performance compared to the MMSE-SIC receiver at BLER $= 0.01$, even when the channel variation is extremely severe, as shown in Fig. 5(b). In addition, for reference, assuming an ideal case with no noise estimation error when performing successive IF, it can be seen that SC-IF outperforms all the other schemes over the entire SNR range. It is also demonstrated that the proposed SC-IF and successive IF schemes still outperform the basic IF scheme when the channel variation is minimal, as shown in Fig. 5(a).

VI. Conclusion

This paper has presented a practical coding scheme for successive IF based on off-the-shelf binary codes, such as turbo or LDPC codes, rather than lattice codes. Then, we designed and introduced an SC-IF scheme using practical binary codes, which effectively combines each advantage of the successive IF and MMSE-SIC schemes into a single MIMO scheme. The proposed SC-IF provides consistent gains over conventional linear receivers for most channel correlation and variation parameters, particularly in environments where there was no apparent winner, such as line-of-sight dominated channel, frequency-selective fading channel, and time-varying fading channel. This paper's findings indicate that the proposed SC-IF scheme has the potential to serve as a viable MIMO coding scheme to support high-throughput services in 5G-Advanced and future generations of cellular networks. Moreover, based on our results, several intriguing new issues can be investigated. For instance, it would be an interesting follow-up study to design a practical coding scheme for multi-user communication based on readily available binary codes by extending our scheme to various multi-user MIMO scenarios.

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